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SEQUENTIAL RESTRUCTURING OF DEBT CLASSES, ABSOLUTE PRIORITY VIOLATION AND SPREAD REVERSALS UNDER CHAPTER 11

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ABSTRACT. Under U.S. Bankruptcy Code, equity holders can restructure different debt classes at a time. Recognizing this allows us to endogenize, in continuous time, not only the restructuring threshold but also the restructuring order of senior and junior classes. Unlike previous studies, sequential restructuring explains absolute priority violation (APV) not just among debt and equity but also among debt classes. The extent of APV leads to positive credit spreads even if senior creditors are fully secured and virtually immune to default risk. Moreover, sequential restructuring can lead to reversals in the credit spreads. We provide sufficient conditions for avoiding reversals.

JEL Classification: G12, G32, G33.

Key Words: strategic debt service, bankruptcy, Nash Bargaining, debt priority structure, geometric Brownian motion.

Since Leland's (1994) seminal work, the valuation of corporate debt has incorporated strategic considerations during renegotiation. Leland endogenizes the restructuring threshold by allowing the payment of promised coupons through additional equity issues until the equity value is driven to zero. A piece of literature on strategic bankruptcy, which includes the work of Leland and Toft (1996), Anderson and Sundaresan (1996), and Mella-Barral and Peraudin (1997), explains absolute priority violation (APV) among debt and equity holders. In turn, APV significantly increases credit spreads.

The implications of these studies may be limited when debt restructuring is governed by a bankruptcy code, however. The role of the bankruptcy code in the allocation of bargaining power is significant in the valuation of renegotiable debt. A few recent studies on strategic bankruptcy accounting for bargaining power considerations (e.g., Mella-Barral and Perraudin, 1997; Hege and Mella-Barral, 2000; Hege and Mella-Barral, 2002; Fan and Sundaresan, 2000; Hennessy, Hackbarth and Leland, 2005) incorporate renegotiation by modelling private workouts where disagreement generally triggers liquidation and the absolute priority rule. Corporate debt valuation models of out-of-court renegotiations provide a useful framework to analyze private workouts. However, as noted by Brown (1989) in a game-theoretical analysis of Chapter 11, a formal bankruptcy proceeding differs from private workouts in a number of dimensions, which affect the values of corporate securities.

Yet, very few pricing theories have explicitly addressed the bankruptcy code as a structural issue in determining credit spreads (Sundaresan, 2000). We attempt to do this by focusing on the U.S. Bankruptcy Code for corporate restructuring. Our model's distinctive feature is the expanded set of strategic actions available to the equity holders under Chapter 11.

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Briefly, a Chapter 11 filing includes two essential rules: (i) the equity holders have the exclusive right to file a first restructuring plan and (ii) those claimholders left 'unimpaired' by a plan[1] lose their veto power. Therefore, in formal bankruptcy, the equity holders can renegotiate with one creditor at a time in a sort of 'private renegotiation' while excluding unimpaired creditors. This possibility greatly enhances the equity holders' strategic behavior in bankruptcy, which concerns not only the timing of bankruptcy but also whether to restructure the junior debt, the senior debt, or both. Consistently with empirical evidence[2], the set of strategic actions is further expanded by the possibility of filing for formal renegotiation a number of times. Therefore, our setting allows for *sequential restructuring* of different debt classes.

We find a unique sequence of equilibrium plans which, depending on the parameters, either restructures the junior creditor first and the senior one later, or vice versa.

In particular, for reasonable level of bankruptcy costs (not excessively high) and junior face value (not negligibly small), it is more likely that the senior debt is not restructured the first time bankruptcy is triggered and will be restructured only if the firm value falls below a certain level, which triggers a new bankruptcy proceeding. This kind of equilibrium is rather common in Chapter 11 reorganizations where, typically, senior secured classes are left unimpaired or, when impaired, the impairment involves all classes (see LoPucki, 1993 and LoPucki 2004).

Most important, we find that this kind of equilibrium may aggravate APV because, by delaying renegotiation with the senior creditor (until the firm value is sufficiently low), the equity holders reduce the liquidation threat of the creditor when renegotiation takes place. In principle, when the firm liquidation value is high enough, a seniority provision might be a valuable asset (because senior claims might recover substantial value even if the firm were liquidated). However, when the senior creditor is not impaired, seniority is not enforceable (only impaired classes retain veto power). No matter how valuable collateral might be, it simply cannot be seized by unimpaired senior classes. Interestingly, while junior classes are always better off in renegotiation rather than liquidation, senior classes may perform better if the firm were liquidated. In other words, the extent of APV in Chapter 11 may generate positive renegotiation premia for senior classes (i.e., the claim liquidation value is above its value in Chapter 11 reorganization).

Also, for sufficiently low senior face value, renegotiation premia may be 100% of the senior credit spread. In this case, positive senior credit spreads are fully explained by sequential renegotiation. This could not be the case in a pure liquidation scenario *à la* Merton (1974), or under a restructuring system such as the UK Insolvency Law which gives senior creditors substantial control over the renegotiation process[3] (see Gower, 1992).

Violation of the absolute priority rule in Chapter 11 reorganizations has been documented by a number of studies (among others see Weiss, 1990, Altman, 1991, Fabozzi et al. 1993, Franks and Torous 1994). In particular, the possibility of positive renegotiation premia to senior creditors is in line with Bebchuck and Fried (1996) who argue that secured creditors may receive less than what they would receive in a Chapter 7 liquidation. Also our result is in line with Pulvino and Pidot (1997) who find that bonds with very high collateral ratios (which, they argue, should be virtually immune to default risk) yield 160 basis points above highly-rated bond yields.

Furthermore, we find that when a plan impairs the senior creditor first and the junior one later on, the senior credit spread may be higher than the junior one. Reversal of the spreads does not occur if the junior creditor is restructured/impaired before the senior creditor. The possibility of spread reversal may have some practical relevance in two circumstances.

First, for reasonable level of bankruptcy costs, reversal can occur only for a particularly low level of junior face value. That is, the reversal is essentially a *reversal between very large claims versus very small unsecured claims*. Reversal would be consistent with the treatment of *convenience claims* (small unsecured claims placed in a separate class for administrative convenience) which in Chapter 11 reorganizations are often unimpaired.

Second, with respect to large reorganizations, involving more complex priority structures, it is often the case that low priority classes, such as senior unsecured and junior (or senior subordinated and junior classes), recover very similar value or junior classes may recover slightly more than more senior unsecured classes (see Altman and Eberhart, 1994; and Franks and Torous, 1994). It is beyond the scope of this paper to extend the model to account for more complex debt structures. However, our predictions are in line with these empirical findings.

Finally, our in-court restructuring can be a benchmark for out-of-court restructuring. Sequential renegotiation justifies the possibility of out-of-court debt forgiveness and strategic default on a single class of claims. Although debt issues often contain cross default provisions, this does not generally prevent private workout and strategic default on a single creditor. Indeed, our equilibrium is consistent with the possibility of out-of-court strategic default on a single class of claims accommodated by other classes of claimants.

The paper is organized as follows. Section I introduces the main assumptions about the value of the firm when it continues operating and when it is liquidated. Section II sets out the bankruptcy rules for renegotiation. In Section III, we present a simpler version of the model, with a single creditor. Section IV describes the model with two classes of debt. In Section V, we summarize and explain our results. In Section VI, we consider the implications of our results on the credit spreads. Section VII concludes.

I. **Firm value: basic assumptions**

The value of the firm is driven by an underlying cash flow process, p_t, which follows a geometric Brownian motion[4] with drift μ and volatility σ. For simplicity, we assume that there are no variable costs and the scrapping value of the firm, γ, is constant. Moreover, agents are risk-neutral[5] and fully informed, with a risk-free interest rate r.

Because the business can be shut down, the value of the firm can be written as

$$V(p_t, \underline{p}) = \frac{p_t}{r-\mu} + \left(\gamma - \frac{\underline{p}}{r-\mu}\right)\left(\frac{p_t}{\underline{p}}\right)^{\lambda} \tag{1}$$

where λ is the negative root of the quadratic equation[6] $r - \mu\lambda - \frac{\sigma^2}{2}\lambda(\lambda-1) = 0$ and is equal to

$$\lambda = \frac{-(\mu - \sigma^2/2) - \sqrt{(\mu - \sigma^2/2)^2 + 2\sigma^2 r}}{\sigma^2}. \tag{2}$$

The value of the firm is maximized when the threshold level for shutting down, $\underline{p}$, is such that

$$\underline{p} = \arg\max V(p_t, \underline{p}), \tag{3}$$

which results into

$$\underline{p} = \frac{\lambda}{\lambda - 1}\gamma(r-\mu). \tag{4}$$

To show that liquidation is generally inefficient as direct bankruptcy costs arise, we assume that the firm can be liquidated at any time and sold to a potential new owner who is as efficient as the original owner. This means that the new owner can generate a value $V(p_t, \underline{p})$ by running a pure equity firm. The liquidation sale of the firm occurs according to a Nash bargaining situation between the initial owner (the equity holders) and the new owner, where the terms of the agreement define the firm sale price, say $V_L(p_t)$, which is therefore the unknown of the bargaining problem. As is well known, the axiomatic solution to the Nash bargaining problem can be found by maximizing the "Nash product," say NP, which is the product of the differences between the agreement and disagreement payoff for each player. In our case, the agreement payoffs are: $V_L(p_t)$ to the initial owner (that is, the sale price agreed on and received by initial owner) and $V(p_t) - V_L(p_t)$ to the new owner[7]. The disagreement payoffs are: γ to the initial owner (i.e., if no agreement is reached, the firm is sold piecemeal at its scrapping value) and zero to the new owner. We do not restrict this bargaining to be symmetric. One can imagine that the initial owner has a relative bargaining power $\alpha \in [0, 1)$ and, hence, the new owner has bargaining power $1 - \alpha$. Given these specifications, the Nash product can be written as

$$NP = (V_L(p_t) - \gamma)^{\alpha}(V(p_t) - V_L(p_t))^{1-\alpha}$$

and it can be easily found that the agreed sale price which maximizes the Nash product is

$$\arg\max NP(V_L(p_t)) = V_L(p_t) = \alpha V(p_t) + (1 - \alpha)\gamma. \tag{5}$$

Therefore, from our bargaining formulation, the liquidation value is a weighted average between $V(p_t)$ and γ, and it is always greater or equal to the scrapping value. This specification of liquidation value, V_L, partly resembles the formulation by Leland (1994) and Leland and Toft[8] (1996) and it converges to their formulation when the scrapping value is equal to zero. Also, our formulation of V_L is in line with Mella-Barral and Perraudin (1997) in that their liquidation value is always below the maximum firm value $V(p_t)$ but never below the scrapping value of the firm. Moreover, their bankruptcy costs become zero as the state variable, p_t, approaches the optimal shutting down trigger, which is also confirmed by our bargaining formulation of V_L. Note, our bankruptcy costs is given by the difference $V(p_t) - V_L(p_t)$, which rearranges as $(1 - \alpha)(V(p_t) - \gamma)$. Hence, the bankruptcy costs converge to zero when p_t tends to the optimal shutting down trigger $\underline{p}$.

Furthermore, we assume that the firm has issued a perpetual debt with face value F. The debt is allocated over two classes of claims–a senior and a junior claim with face values respectively $F_s = b_s/r$ and $F_j = b_j/r$ (and $F_s + F_j = F$) where b_s and b_j are the contractual coupon payments. Also, to make the problem interesting, we assume that the senior claim face value is such that[9] $F_s > \gamma$.

II. **Bankruptcy rules**

Financial distress is often accompanied by formal renegotiations and rarely by liquidations. As reported by Franks and Torous (1989), after the introduction of the 1978 U.S. Bankruptcy Act, the number of firms seeking bankruptcy protection has increased tremendously.

The Bankruptcy Code has been amended a number of times, with most recent amendments set out with "The Bankruptcy Abuse Prevention and Consumer Protection Act of 2005" (shortly, the 2005 Act) effective since October 17, 2005. Below, we stylize Chapter 11 rules by setting out our main assumptions on the formal bankruptcy process[10]. The 2005 Act does not have any impact on our setting, however.

1. Timing of bankruptcy. As in the strategic bankruptcy literature (e.g., Leland, 1994, Mella-Barral and Perraudin, 1997) and consistently with Chapter 11, we assume that the firm can voluntarily enter bankruptcy by ceasing to pay the contractual coupon[11]. Strategic bankruptcy implies that the equity holders are free to issue new equity to cover operating losses. As soon as the firm stops meeting its contractual obligation a formal renegotiation procedure is triggered.

2. First proposal and impairment rule. Following Chapter 11, we assume that the equity holders have an exclusive right to propose a first reorganization plan[12], which must be approved by all claimants. Precisely, in Chapter 11, approval of two-thirds majority within each class is required, but to keep the bargaining simple and abstract from holdout problems, we treat each class of claimants as a single agent.

Also as in Chapter 11, we assume that a creditor cannot reject a plan if he receives cash equal to the face value of his claim or if the plan calls for no scaling down of the coupon payment scheduled in the existing contract. In this case the creditor is said to be unimpaired[13] by the plan and loses his veto power[14]. The 'impairment' rule allows equity holders to negotiate with one creditor at a time.

In other words, at this first stage of renegotiation, the equity holders have the right to make a take-it-or-leave-it offer, which impairs one or both creditors. If this offer is accepted by the impaired creditors (the unimpaired ones lose veto power), the plan is confirmed, the game ends and restructuring proceeds. This initial stage is referred to as 'private game'.

Note that, we have implicitly assumed that only consensual plans (i.e. plans accepted by impaired creditors) are confirmed. However, in principle, Chapter 11 allows for confirmation by the Court of non-consensual plans. That is, on request of the plan's proponent, the Court may confirm a plan in spite of rejection of some impaired classes as long as: i) at least one impaired class has accepted the plan and ii) impaired rejecting classes receive at least what they would receive in a Chapter 7 liquidation[15]. As reported by Lopucki and Whitford (1990), non-consensual plans –so called 'crammed-down' plans– are rarely the outcome of restructuring (in a sample of 43 Chapter 11 reorganizations, in no case was a plan confirmed without approval of all debt classes). Not even seems cram-down to be a strategic threat to speed up acceptance of a plan. According to Lopucki and Whitford, bankruptcy practitioners rarely suggest a cram-down strategy. Rather than for strategic reason, they argue that cram-down is accounted for in Chapter 11 because when a class receives nothing under a plan, that class is deemed to reject –even if no actual disagreement arises– and no vote from that class is taken (Bankruptcy Code, Section 1126, Paragraph (g)). Hence, in such a scenario, cram-down is necessary as a practical expedient to avoid non-confirmation of plans which are unanimously agreed on.

3. Subsequent proposals. We assume that without time delay[16], if the first proposal is rejected, the renegotiation moves to a second stage in which any player may file competing plans.

There are no specific agenda rules concerning subsequent proposals in the bankruptcy code. Once the equity holders' proposal has been rejected, the 'rules' of the game become unbiased toward different players. In Chapter 11, when multiple plans are filed and accepted by the voting classes, the Court should decide which plan to confirm on the ground of 'the preferences of creditors and equity security holders' (Bankruptcy Code, Section 1129, Paragraph (c)). A certain degree of discretion seems to be granted at this stage and the outcome of the negotiation depends on the ability of the players to propose reorganization plans and influence the court.

There are many different ways of modeling the second stage of the renegotiation[17]. Unlike Brown (1989), we recognize the limits of a 'refereed' bargaining system. As argued by legal scholars, judicial discretion is granted in many circumstances (see A. Schwartz, 2002). Therefore, our bargaining setting accounts for exogenous asymmetries between parties which might reflect the ability of claimholders to influence the court. As with Welch (1997), such asymmetry can be explained as a reflection of different organization skills and reputation benefits[18].

We use a simple framework that highlights the bargaining power of the different players and, as with Fan and Sundaresan (2000), we assume that a Nash axiomatic solution is enforced by the court[19]. Therefore, players split the gains generated from restructuring according to their bargaining power. Also, in line with Aivazian and Callen (1983) a cooperative approach seems to best fit a formal renegotiation supervised by the Court. Moreover, the Nash axiomatic approach is easy to handle, extends unchanged to multilateral bargaining situations. When it is believed that there are no asymmetries at this stage of the negotiation process, then the bargaining power of players can be simply set equal to 1/3.

If no agreement is reached at this stage, the firm goes into liquidation without delay; therefore, the disagreement payoffs in the Nash bargaining correspond to the liquidation payoff of each claimant.

More formally, let $i = e, s, j$ denote the equity holders (the senior and the junior creditor) respectively. As is well known[20], in the Nash axiomatic bargaining, player 'i' can guarantee her disagreement payoff (i.e., liquidation payoff), L_i, plus a share of the surplus, $V(p_t) - V_L(p_t)$, which depends on the relative bargaining power. Therefore, in the Nash bargaining, player 'i''s claim value is equal to

$$\underline{P}_i = \xi_i(V(p_t) - V_L(p_t)) + L_i, \tag{6}$$

where ξ_i and L_i denote the bargaining power and the liquidation payoff of player 'i' respectively. We will refer to this stage as the 'joint game'.

4. Liquidation. If the firm is liquidated, the absolute priority rule (APR) applies. Formally, under the APR, the liquidation payoffs for s, j and e are respectively

$$\begin{aligned} L_s &= \min\{F_s, V_L\} && (7) \\ L_j &= \min\{F_j, V_L - L_s\} && (8) \\ L_e &= \max\{V_L - L_s - L_j, 0\}. && (9) \end{aligned}$$

5. Terms of Contracts and Enforceability. In practice, the renegotiation can be implemented by reducing coupon payments while the firm is in a default region through a variable, state contingent, debt service flow as in Mella-Barral and Perraudin (MBP) (1997). Therefore we focus on restructuring plans which consist of piecewise right-continuous service flow function of the state variable p_t, denoted as $b_i(p_t)$ for $i = s, j$ with $b_i(p_t) \leq b_i$ (the equality holds if class i is unimpaired under the plan).

Because the equity holders cannot commit to remaining out of bankruptcy and can trigger renegotiation at any time, creditors will underwrite a state contingent contract only if this is self-enforceable on the equity side (in the sense that the equity holders will not have incentive to deviate). It is reasonable, then, to imagine that players bargain over short-term contracts, i.e., instantaneous payoffs, and that they can renegotiate continuously. Though this might not be realistic, a continuous time renegotiation provides the necessary benchmark to derive a self-enforceable contract.

Let the set of plans proposed by the equity holders in the "private game" be $\mathcal{P} = \{P_e, P_j, P_s : P_e + P_j + P_s = V(p_t)\}$. At the second stage, "joint game," each player can guarantee a payoff, denoted as $\underline{P}_i$ for $i = e, j, s$ which depends on the bargaining strength of players. Therefore, a reorganization plan will be accepted at the first round by an 'impaired claimant,' denoted by 'i,' only if $P_i \geq \underline{P}_i$.

Panel A shows the two-step game in which, at the first round, the equity holders choose whether to impair:(i) the junior creditors by proposing a plan $\mathcal{P}_j \subset \mathcal{P}$ where $\mathcal{P}_j = \{P_e, P_j, P_s : P_e \geq \underline{P}_e, P_j \geq \underline{P}_j, P_s < \underline{P}_s\}$, (ii) the senior creditor by proposing a plan $\mathcal{P}_s \subset \mathcal{P}$ where $\mathcal{P}_s = \{P_e, P_j, P_s : P_e \geq \underline{P}_e, P_j < \underline{P}_j, P_s \geq \underline{P}_s\}$ or (iii) both the senior and the junior creditors and in this case the only feasible plan, say $\mathcal{P}_{s,j} \subset \mathcal{P}$, must guarantee that each player receives his reservation payoff in the joint game, i.e. $\mathcal{P}_{s,j} = \{P_e, P_j, P_s : P_e = \underline{P}_e, P_j = \underline{P}_j, P_s = \underline{P}_s\}$.

Panel A: Bankruptcy game

e offers:	on rejection by:	i=e,j,s, share	on rejection	Liquidation
$\mathcal{P}_j$ impairing j,	j	the 'surplus'	by at least	payoffs L_i, for
or $\mathcal{P}_s$ impairing s,	s	according to a	one player	$i = e, j, s$ and
or $\mathcal{P}_{s,j}$ impairing	at least one player	payoff $\underline{\mathrm{P}}_i$.	$\longrightarrow$	$\sum L_i = V_L$
both.	$\longrightarrow$			

III. **A simplified scenario: one creditor only**

We begin by considering a simplified scenario in which the firm has issued only one class of claims, with face value $F_c = b_c/r > \gamma$. At the end of this section, we show that this scenario can also be interpreted as an hypothetical situation where a firm, with different classes of claims (with overall face value F_c), is not allowed to restructure claims sequentially, but only all classes at once.

With only one creditor, the private game is redundant of course as well as the option to impair certain classes. Hence, most of Chapter 11's rules do not play a crucial role in this case. In fact, most of the literature on debt restructuring focuses on private workouts when there is a single class of claims rather than on formal bankruptcy. In our formal bankruptcy setting, renegotiation simply resolves in a Nash bargaining. In fact, given the equilibrium partition in the joint game, (i) the claim holder accepts the first equity holders' proposal only if he receives at least the equilibrium share of the joint game and (ii) the equity holders will offer at most the creditor's equilibrium share of the joint game. Because there is no time delay between the private and joint game, the equilibrium shares in the private and the joint game are the same.

Hypothesis 1A. Assume that there exists a trigger strategy, such that when the state variable p_t crosses a certain trigger, say p_c, the equity holders renegotiate with the debt holder in a Nash bargaining (through a service flow $b_c(p_t) < b_c$).

The equity holders's objective is to maximize the equity value, therefore the threshold level p_c is selected according to the following

$$\begin{aligned} &\max_{p_c} E(p_t, p_c) \\ &\text{st. } E(p_t, p_c) \quad = \quad V(p_t) - C(p_t, p_c) \\ &\qquad\qquad C \quad < \quad F_c, \end{aligned} \tag{10}$$

where $C(p_t, p_c)$ is the debt value for $p_t \geq p_c$.

Hypothesis 2A. When the state variable is in the range $[\underline{p}, p_c]$ we assume that $V_L < F_c$. Because APR applies, this implies that $L_e = 0$ and $L_c = V_L$.

Renegotiation unfolds through a Nash bargaining where, by equation (6) and Hypothesis 2A, the shares to the equity holders and the creditor become respectively

$$\underline{\mathrm{P}}_e = \xi_e(V - V_L) \tag{11}$$

$$\underline{\mathrm{P}}_c = \xi_c(V - V_L) + V_L \tag{12}$$

$$\text{with} \qquad \begin{cases} \xi_i \in [0, 1] \\ \sum_i \xi_i = 1 \end{cases} \quad \text{for} \quad i = e, c.$$

Because the renegotiated shares are already defined by equations (11) and (12), left to determine are the trigger p_c and the service flow $b_c(p_t)$.

To derive the bankruptcy trigger p_c, one can solve the equity holders' maximization problem, or equivalently, the minimization problem:

$$\begin{array}{lccc} \min\limits_{p_c} & C(p_t, p_c) & & \\ \text{st.} & C < F_c & & \\ \text{with} & C(p_t) = \underline{\mathrm{P}}_c & \text{for} & p_t = p_c \end{array} \tag{13}$$

where the debt value, $C(p_t, p_c)$ can be written as

$$C(p_t) = \frac{b_c}{r} + \left(\underline{\mathrm{P}}_c(p_c) - \frac{b_c}{r}\right)\left(\frac{p_t}{p_c}\right)^{\lambda}. \tag{14}$$

Solving the above problem yields the optimal trigger

$$p_c^* = \frac{\lambda}{\lambda - 1}\frac{b_c/r - \gamma(1-\alpha_{\xi_c})}{\alpha_{\xi_c}}(r-\mu), \tag{15}$$

with

$$\alpha_{\xi_c} = \xi_c(1-\alpha) + \alpha. \tag{16}$$

Moreover, as shown in Appendix, the trigger p_c^* guarantees that Hypothesis 2A holds (because of the similarity in the calculation, we prove this along the proof of Hypothesis A in Appendix).

Last, the service flow function $b_c(p_t)$ can be derived as follows. Under risk neutrality, the claim value $C(p_t)$ is free of arbitrage opportunity if and only if $b_c(p_t)$ solves the differential equation[21]

$$rC(p_t) = b_c(p_t) + \mu p_t C'(p_t) + \frac{\sigma^2}{2}{p_t}^2 C''(p_t), \tag{17}$$

where

$$\begin{array}{lcl} b_c(p_t) = b_c & \text{if} & p_t \in (p_c^*, \infty) \\ b_c(p_t) < b_c & \text{if} & p_t \in [\underline{p}, p_c^*] \end{array}$$

and $C(p_t)$ satisfies: i) the smooth-pasting conditions $C(p_c^*) = \underline{\mathrm{P}}_c(p_c^*)$ and $\frac{\partial C(p_t)}{\partial p_t}\mid_{p_c^*} = \frac{\partial \underline{\mathrm{P}}_c(p_t)}{\partial p_t}\mid_{p_c^*}$ (i.e., continuity in level and first derivative) and ii) the no-bubbles conditions $\underline{\mathrm{P}}_c(\underline{p}) = \gamma$ and $\lim_{p_t\to\infty} C(p_t) = \frac{b_c}{r}$.

One can find that the debt value, $C(p_t)$, defined by equation (14), satisfies all the above conditions. In particular, equation (14) is continuous in level and satisfies the no-bubble conditions by construction. Also, continuity in the first derivative is guaranteed by the first order condition of the equity holders' maximization problem, that is, $\partial C(p_t, p_c)/\partial p_c = 0$.

Substituting for C' and C'' into equation (17) delivers the unique solution, $b_c(p_t)$, given by

$$b_c(p_t) = \begin{cases} b_c & \text{if} \quad p_t \in (p_c^*, \infty) \\ \alpha_{\xi_c} p_t + (1-\alpha_{\xi_c}) r\gamma & \text{if} \quad p_t \in [\underline{p}, p_c^*] \end{cases}$$

with α_{ξ_c} defined in equation (16).

Most important, one can interpret this 'simplified scenario' as a benchmark where a firm with two classes of debt (a senior and a junior class with total face value $F_c = F_s + F_j$) can only restructure all claims at once (that is, sequential restructuring is not allowed and renegotiation starts with the joint game). In this case, the debt value C is the sum of senior and junior debt values (S and J respectively, therefore $C = S + J$) and equation (12) represents the sum of the senior and junior shares in the Nash bargaining ($\underline{\mathrm{P}}_s$ and $\underline{\mathrm{P}}_j$ respectively) as defined in equation 6 (with $\xi_c = \xi_s + \xi_j$).

An important difference from the case of a single creditor is the following. While Hypothesis 2A always holds ($V_L < F_c$) at $p_t \leq p_c$, it is not guaranteed that also $V_L < F_s$ at p_c. That is, depending on the allocation of face value among creditors it could well be the case that $F_s \leq V_L(p_c) < F_c$ (intuitively, note that p_c, and in turn $V_L(p_c)$, depend on the overall face value F_c and not on the allocation among F_s and F_j). The point is that, if the debt structure is such that $F_s \leq V_L(p_c)$, renegotiating only via temporary reduction of coupon payments is not feasible because the senior creditor can guarantee a renegotiated value $\underline{\mathrm{P}}_s = \xi_s(V - V_L) + F_s$ which is greater than his face value. We do not attempt to solve this problem, however it is clear that a senior debt value $\underline{\mathrm{P}}_s > F_s$ can only be attained by proposing a *new* debt contract – e.g., new face value and temporary concessions (see Mella-Barral (1999))– rather than only *temporary state contingent* concessions $b_s(p_t) < b_s$.

As shown below, when instead sequential restructuring is allowed, as in Chapter 11, debt classes can always be restructured through temporary reduction of cash flow claims[22].

IV. **Sequential restructuring with two creditors**

Our model shows how renegotiation is implemented when the firm has issued two classes of claims as defined in Section I. In this scenario, none of the bankruptcy rules described in Section II are redundant. On the contrary, our bankruptcy framework defines a unique equilibrium in terms of renegotiating strategy and restructuring plan.

Hypothesis 1 Assume that there exists a strategy in terms of stopping times, such that (i) when the state variable p_t crosses a certain trigger, say p_j, the equity holders start renegotiating with the junior creditor while $p_t < p_j$ (by proposing a service flow $b_j(p_t) < b_j$), and (ii) when p_t crosses the trigger p_s they start renegotiating with the senior creditor (through a service flow $b_s(p_t) < b_s$).

Now, let $max\{p_j, p_s\} = p^*$ and $min\{p_j, p_s\} = p_*$. The equity holders' strategy can be shortly summarized as follows:

$$\text{when} \quad p_* < p_t \leq p^* \text{ propose plan } \mathcal{P}_i,\ b_i(p_t) \text{ with } \begin{cases} i = j \text{ if } p^* = p_j \\ i = s \text{ otherwise} \end{cases} \tag{18}$$

$$\text{when} \quad \underline{p} \leq p_t \leq p_* \text{ propose plan } \mathcal{P}_{s,j}, \begin{cases} b_j(p_t) \\ b_s(p_t) \end{cases} \tag{19}$$

$$\tag{20}$$

Therefore, the problem of the equity holders can be generalized as

$$\max_{p_j, p_s} E(p_t) \tag{21}$$

$$\begin{aligned} \text{st. } E(p_t) &= V(p_t) - S(p_t, p_s, p_j) - J(p_t, p_j, p_s) \\ S &< F_s \\ J &< F_j, \end{aligned}$$

where $E(p_t)$, $S(p_t)$ and $J(p_t)$ denotes the values of equity, senior and junior debt and F_s, F_j are the face values.

Hypothesis 2 We assume that $V_L < F_s$ when the state variable is in the range $[\underline{p}, p_s]$. As absolute priority applies, this implies that $L_e = 0, L_j = 0$ and $L_s = V_L$, therefore the shares in the joint game will be

$$\underline{\mathrm{P}}_e = \xi_e(V - V_L) \tag{22}$$

$$\underline{\mathrm{P}}_j = \xi_j(V - V_L) \tag{23}$$

$$\underline{\mathrm{P}}_s = \xi_s(V - V_L) + V_L \tag{24}$$

$$\text{with} \qquad \begin{cases} \xi_i \in I = [0, 1] \\ \sum_i \xi_i = 1 \end{cases} \qquad \text{for} \quad i = e, j, s.$$

A. *Equilibrium offers and trigger strategy*

Next we determine the equilibrium values, E, J, and S, and the optimal trigger strategy $\{p_j, p_s\}$. We already know, by equations (22)-(24), the values of equity, senior,and junior claims when[23] $\underline{p} \leq p_t \leq p_*$; therefore, the problem is to determine E, J, and S when $p_* < p_t \leq p^*$.

As shown in the following proposition, we characterize an impaired claim value under an accepted offer from the equity holders in the private game.

Proposition 1. *When $p_* < p_t \leq p^*$, the smallest offer by the equity holders to the junior creditor is accepted if and only if $J(p_t, p_j) + S(p_t, p_s) = \underline{P}_j + \underline{P}_s$.*

Proof Let us define an arbitrage strategy. In the private game, when $p_* < p_t \leq p^*$, after an equity holders' offer, the impaired creditor can: 1) buy the unimpaired claim at $S(p_t, p_s)$ or $J(p_t, p_j)$, 2) reject the equity holders' plan (therefore losing $J(p_t, p_s)$ or $S(p_t, p_s)$), and 3) before the joint game starts, sell the unimpaired claim at $\underline{P}_s(p_t)$ or $\underline{P}_j(p_t)$. The payoff from such a strategy, say Π, is given by

$$\Pi = \underline{P}_j + \underline{P}_s - J(p_t, p_j) - S(p_t, p_s). \tag{25}$$

Therefore, an offer from the equity holders is accepted by an impaired creditor, if and only if $\Pi \leq 0$, that is $J(p_t, p_s) + S(p_t, p_s) \geq \underline{P}_j + \underline{P}_s$. Also, as shown in Appendix, $\Pi \geq 0$ under an equity holders' proposal. Then we conclude that the smallest offer which is accepted by the impaired creditor must be such that $\Pi = 0$, which yields

$$J(p_t, p_j) + S(p_t, p_s) = \underline{P}_j + \underline{P}_s. \tag{26}$$

■

Now we can find the triggers p_j and p_s under two possible strategies: CASE A) $p_s < p_j$, i.e. the equity holders impair the junior first, or CASE B) $p_s \geq p_j$, the senior creditor is impaired first (if $p_s > p_j$) or jointly with the junior creditor (for $p_s = p_j$). Then, we show that a unique equilibrium strategy exists.

CASE A: $p_s < p_j$**.** The equity holders impair the junior creditor only, when $p_s < p_t \leq p_j$, and also the senior creditor when $\underline{p} < p_t \leq p_s$.

The triggers p_s and p_j can be derived by working backward. When p_t is greater than p_s, the senior value is equal to

$$S(p_t, p_s) = \frac{b_s}{r} + \left(\underline{P}_s(p_s) - \frac{b_s}{r}\right)\left(\frac{p_t}{p_s}\right)^{\lambda}, \tag{27}$$

which, minimized with respect to p_s gives[24]

$$p^*_{s2} = \frac{\lambda}{\lambda - 1}\frac{b_s/r - \gamma(1-\alpha_{\xi_s})}{\alpha_{\xi_s}}(r - \mu), \tag{28}$$

with

$$\alpha_{\xi_s} = \xi_s(1-\alpha) + \alpha. \tag{29}$$

Then, minimising the junior value with respect to p_j, which, by equation (26) (Proposition 1) and (28), is equal to

$$J(p_t, p_j) = \frac{b_j}{r} + \left\{\underline{P}_j(p_j) + \underline{P}_s(p_j) - S(p_j, p^*_{s2}) - \frac{b_s}{r}\right\}\left(\frac{p_t}{p_j}\right)^{\lambda}, \tag{30}$$

yields the optimal trigger

$$p^*_{j1} = \frac{\lambda}{\lambda - 1} \frac{(b_s + b_j)/r - \gamma(1 - \alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}}(r - \mu), \tag{31}$$

with

$$\alpha_{\xi_{s,j}} = (\xi_s + \xi_j)(1 - \alpha) + \alpha. \tag{32}$$

CASE B: $p_s \geq p_j$**.** The equity holders impair the senior creditor only, when $p_j < p_t \leq p_s$, and also the junior creditor when $\underline{p} < p_t \leq p_j$.

As before, by working backward, one can find p_j first and then p_s. Minimizing the junior value, which, at $p_t > p_j$ is equal to

$$J(p_t, p_j) = \frac{b_j}{r} + \left(\underline{\mathrm{P}}_j(p_j) - \frac{b_j}{r}\right)\left(\frac{p_t}{p_j}\right)^{\lambda}, \tag{33}$$

gives

$$p^*_{j2} = \frac{\lambda}{\lambda - 1} \frac{b_j/r + \gamma\xi_j(1 - \alpha)}{\xi_j(1 - \alpha)}(r - \mu). \tag{34}$$

Then, by minimizing the senior value with respect to p_s, which (by equation (26) –Proposition 1, and equation (34)), when $p_t > p_s$, is equal to

$$S(p_t, p_s) = \frac{b_s}{r} + \left\{\underline{\mathrm{P}}_j(p_s) + \underline{\mathrm{P}}_s(p_s) - J(p_s, p^*_{j2}) - \frac{b_s}{r}\right\}\left(\frac{p_t}{p_s}\right)^{\lambda}, \tag{35}$$

and solving for the optimal trigger yields

$$p^*_{s1} = \frac{\lambda}{\lambda - 1} \frac{(b_s + b_j)/r - \gamma(1 - \alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}}(r - \mu), \tag{36}$$

with $\alpha_{\xi_{s,j}}$ defined in equation (32).

B. *Unique Equilibrium strategy*

The equilibrium strategy is unique if, once the equity holders impair creditor $i = s$ or j (leaving creditor $\underline{\mathrm{i}} \neq i$ unimpaired), they will not change strategy in the future (that is, impairing $\underline{\mathrm{i}} \neq i$ and paying the full contractual coupon to creditor i).

Regardless of the fact that the creditor who is impaired first is the junior (as in CASE A) or the senior one (CASE B), the higher bankruptcy trigger does not change. In fact, $p^*_{j1} = p^*_{s1}$.

Let us denote with $\bar{p}$ the trigger $p^*_{j1} = p^*_{s1}$. One can find, by some simple algebra, that

$$\begin{array}{lll} p^*_{s2} < \bar{p} & \text{if and only if} & \frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F - \gamma \\ p^*_{j2} \leq \bar{p} & \text{if and only if} & \frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} \leq F - \gamma. \end{array} \tag{37}$$

These two inequalities allow us to state an important result in the following proposition.

Proposition 2. *The optimal impairment strategy is unique and the equity holders restructure claims sequentially according to the trigger strategy*

$$\begin{array}{lll} \{p^*_{j1}, p^*_{s2}\} & \textit{iff} & \frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F - \gamma \\ \{p^*_{s1}, p^*_{j2}\} & \textit{iff} & \frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} \leq F - \gamma. \end{array} \tag{38}$$

Proof When $p^*_{s2} < \bar{p}$ (i.e. $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F - \gamma$) is also true that $p^*_{j2} > \bar{p}$, but p^*_{j2} is an optimal trigger only if $p^*_{j2} \leq p^*_{s1} = \bar{p}$. Therefore, we can conclude that p^*_{j2} does not exist in this case and hence there is a unique strategy left, that is $\{p^*_{j1}, p^*_{s2}\}$. Under this strategy the equity holders impair the junior creditor first when $p_t = \bar{p}$ and senior later, when $p_t = p^*_{s2}$.

If instead $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} \leq F - \gamma$ then $p^*_{j2} \leq \bar{p}$ and $p^*_{s2} > \bar{p}$. Nevertheless, for p^*_{s2} to be optimal it must be the case that $p^*_{s2} \leq p^*_{j1} = \bar{p}$. As with the previous case, we conclude that p^*_{s2} does not exist and there is only one strategy left corresponding to $\{p^*_{j2}, p^*_{s1}\}$. Therefore, the equity holders impair the senior creditor first (i.e., when $p_t = \bar{p}$) and, when $p_t = p^*_{j2}$, the junior creditor also. ■

Also, as shown in Appendix, under both strategies, $\{p^*_{s2}, p^*_{j1}\}$ and $\{p^*_{s1}, p^*_{j2}\}$, Hypothesis 2 holds, that is $V_L(p_s) < F_s$ (with p_s equal to either p^*_{s2} or p^*_{s1}). Unlike the case depicted in Section III where only joint restructuring is allowed, this guarantees that the senior creditor is not involved in the restructuring when he could guarantee his full face value in liquidation.

Comparing sequential renegotiation with joint renegotiation (depicted in Section III), even though the overall debt value and the higher threshold $\bar{p}$ are the same under both restructuring systems[25], however, the valuation of individual debt classes under the two systems differs substantially. We will discuss this point more extensively in Section VI.

Finally, the existence of a unique equilibrium in formal bankruptcy provides players' reservation payoffs in private workouts. As formal bankruptcy is obviously a benchmark for out-of-court restructuring, one may note that single-debt-class restructuring is consistent with the fact that private workouts are often targeted to single classes of claims, and that cross default provisions do not prevent a private renegotiation. In this regard, it is worth to mention that in formal bankruptcy, if a plan of reorganization is confirmed, even if a cross-default can be asserted, confirmation of the plan resolves and eliminates it. That is, when non-impaired debt contains cross-default provisions, cure of the cross-defaults can be accomplished even if other classes are impaired under the plan (see in In re: Mirant Corporation, et al., Bankruptcy Court, Northern District of Texas, case n. 03-46590-DML-11, 2005). This point is obvious because it would be inconsistent with the purposes of Chapter 11 to allow cross-defaults to defeat confirmation.

C. *Restructuring plan*

Depending on the set of parameters the equilibrium trigger strategy is either $\{p^*_{s2}, p^*_{j1}\}$ (if $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} \leq F - \gamma$) or $\{p^*_{j2}, p^*_{s1}\}$ (if else) with corresponding equilibrium values, $J(p_t)$ and $S(p_t)$, defined in CASE A and CASE B (of Section A) respectively. Therefore, according to the two possible sets of parameters, two corresponding restructuring plans are defined in the following propositions. Our derivation of the debt service flow functions follows the same line as the derivation of $b_c(p_t)$ in Section III. Therefore, we only present our results and refer the reader to the Appendix for the derivation.

Proposition 3. *If* $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F - \gamma$ *the equilibrium restructuring plans impair the junior and senior creditor according to the service flow functions*

$$b_j(p_t) = \alpha_{\xi_{s,j}} p_t + (1-\alpha_{\xi_{s,j}})\gamma r - s_s(p_t) \qquad \mathit{for} \qquad p_t \leq p^*_{j1} \tag{39}$$

$$b_s(p_t) = \begin{cases} b_s & \mathit{for} \quad p^*_{s2} < p_t \\ \alpha_{\xi_s} p_t + (1-\alpha_{\xi_s})\gamma r & \mathit{for} \quad p_t \leq p^*_{s2} \end{cases} \tag{40}$$

with $\alpha_{\xi_{s,j}}$ *and* α_{ξ_s} *defined by equations (32) and (29) respectively.*

Proof See Appendix. ■

Proposition 4. *If* $\frac{b_j}{r}(\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)}) \leq F-\gamma$ *the equilibrium restructuring plans impair the junior and senior creditor according to the service flow functions*

$$b_s(p_t) = \alpha_{\xi_{s,j}} p_t + (1-\alpha_{\xi_{s,j}})\gamma r - s_j(p_t) \quad \text{for} \quad p_t \leq p^*_{s1} \tag{41}$$

$$b_j(p_t) = \begin{cases} b_j & \text{for} \quad p^*_{j2} < p_t \\ \xi_j(1-\alpha)(p_t - \gamma r) & \text{for} \quad p_t \leq p^*_{j2} \end{cases} \tag{42}$$

Proof See Appendix. ■

V. **Summary**

A) Timing of bankruptcy and impairment strategy. The equity holders trigger bankruptcy and file a plan $\mathcal{P}_i$ or $\mathcal{P}_{s,j}$ according to the following equilibrium strategy:

$$\text{when} \quad p^*_{i2} < p_t \leq \bar{p} \quad \text{file plan } \mathcal{P}_{\neq i}, \text{ with} \begin{cases} i = s & \text{if } \frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F-\gamma \\ i = j & \text{otherwise} \end{cases} \tag{43}$$

$$\text{when} \quad \underline{p} \leq p_t \leq p^*_{i2} \quad \text{file plan } \mathcal{P}_{s,j}, \tag{44}$$

where

$$p^*_{s2} = \frac{\lambda}{\lambda-1}\frac{F_s - \gamma(1-\alpha_{\xi_s})}{\alpha_{\xi_s}}(r-\mu), \tag{45}$$

$$p^*_{j2} = \frac{\lambda}{\lambda-1}\frac{F_j + \gamma\xi_j(1-\alpha)}{\xi_j(1-\alpha)}(r-\mu), \tag{46}$$

$$\bar{p} = p^*_{j1} = p^*_{s1} = \frac{\lambda}{\lambda-1}\frac{F - \gamma(1-\alpha_{\xi_{s,j}})}{\alpha_{\xi_{s,j}}}(r-\mu), \tag{47}$$

and

$$\alpha_{\xi_s} = \xi_s(1-\alpha) + \alpha, \tag{48}$$

$$\alpha_{\xi_{s,j}} = (\xi_s + \xi_j)(1-\alpha) + \alpha. \tag{49}$$

Given the overall face value of the debt, F, the higher bankruptcy trigger is independent of the priority structure of claims. Bankruptcy is triggered the first time, as soon as the state variable crosses the threshold level $\bar{p}$. In other words, given the overall creditors' bargaining power and the face value F, the default region $[\underline{p}, \bar{p}]$ is independent of the type of creditor impaired first and is not affected by the allocation of debt among creditors. To highlight the irrelevance of the debt priority structure over the default region, one can compare two firms which differ only in their debt allocation. Imagine, for instance, the limiting case of an identical firm (where the equity holders have the same bargaining power, ξ_e) with just one creditor with a claim of face value $F = F_j + F_s$ and bargaining power $\xi_c = 1 - \xi_e = 1 - \xi_s - \xi_j$. As shown in Section III, renegotiation starts when p_t crosses $\frac{\lambda}{\lambda-1}\frac{F-\gamma(1-\alpha_{\xi_c})}{\alpha_{\xi_c}}(r-\mu)$ which coincides with the optimal trigger $\bar{p}$ (because $\alpha_{\xi_c} = \alpha_{\xi_{s,j}}$).

Therefore, what defines the default region is the level – not the allocation – of F and the overall bargaining power of creditors vis-a-vis the equity holders. The higher F and/or $\xi_e = 1 - \xi_j - \xi_s$, the earlier bankruptcy occurs.

Given the overall face value, the priority structure of claims determines the default regions of each single claim. In particular, depending on the claims face values, there are two possible default regions for each debt class: (i) $[\underline{p}, \bar{p}]$ and $[\underline{p}, p^*_{s2}]$, respectively for the junior and the senior, if $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F-\gamma$, or (ii) $[\underline{p}, \bar{p}]$ and $[\underline{p}, p^*_{j2}]$, if instead $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} \leq F-\gamma$. When

$\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} = F-\gamma$, $p^*_{j2} = p^*_{s2} = \bar{p}$, that is claims are jointly impaired and the two default regions are the same.

The intuition behind this result is simple. The equity holders always start renegotiating as soon as p_t falls below $\bar{p}$. When the state variable crosses this trigger, they would benefit from reducing coupon payments to the creditor whose face value is relatively high compared to the overall face value F. Therefore, for instance, given F, if, b_j/r is high enough, the equity holders would benefit from impairing the junior creditor first. However, the benefit of impairing the creditor with higher face value must be weighed against the 'strength' of that creditor at the negotiating table; that is, the ability to extract a valuable package of concessions in renegotiation. This depends on the liquidation value of the firm, the priority of the claim, and the creditor's bargaining power. The higher α and γ, the bigger the liquidation value, which, in turn, strengthens the bargaining position of the senior creditor while weakening that of the junior creditor[26]. Therefore, when the liquidation value is sufficiently high, the equity holders impair the junior creditor, who can extract smaller concessions than the senior claimant. The argument is similar when the bargaining power of the junior creditor is sufficiently small. If this is the case, again, the equity holders impair first the junior creditor, that is, the 'weak' player. All these factors are captured by the inequality $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} \gtreqless F-\gamma$, which determines the order in which claims are impaired. For instance, it is more likely that $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} > F-\gamma$ (and hence the junior creditor is impaired first) for high levels of b_j/r, α and γ and low level of ξ_j, consistent with our explanation.

Our argument can be easily summarized by rearranging the inequality $\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} \gtreqless F-\gamma$ as

$$x_s = \frac{\alpha_{\xi_s}}{F_s-\gamma} \gtreqless \frac{\xi_j(1-\alpha)}{F_j} = x_j. \tag{50}$$

where x_s and x_j measure the intensity of the effective bargaining strength of the senior and junior creditor respectively. The terms α_{ξ_s} and $\xi_j(1-\alpha)$ represent the *effective* bargaining strength of creditors (because they account for the exogenous bargaining power, ξ_i, and α, which determines the disagreement/liquidation payoff). Moreover, in equation (50), the effective bargaining strength is measured in units of unsecured face value (that is, $F_s-\gamma$ and F_j). We conclude that the equity holders impair first the creditor with lower intensity of effective bargaining strength.

B) Equity and claims values. According to the trigger strategy in equation (50), if $x_j < x_s$, the junior creditor is restructured first and debt values are equal to:

$$S(p_t) = \begin{cases} \frac{b_s}{r} + \left(\underline{\mathrm{P}}_s(p^*_{s2}) - \frac{b_s}{r}\right)\left(\frac{p_t}{p^*_{s2}}\right)^{\lambda} & \text{if} \quad p_t > p^*_{s2} \\ \underline{\mathrm{P}}_s(p_t) & \text{if} \quad p_t \le p^*_{s2} \end{cases} \tag{51}$$

$$J(p_t) = \begin{cases} \frac{b_j}{r} + \left(\underline{\mathrm{P}}_j(\bar{p}) + \underline{\mathrm{P}}_s(\bar{p}) - S(\bar{p}) - \frac{b_j}{r}\right)\left(\frac{p_t}{\bar{p}}\right)^{\lambda} & \text{if} \quad p_t > \bar{p} \\ \underline{\mathrm{P}}_j(p_t) + \underline{\mathrm{P}}_s(p_t) - S(p_t) & \text{if} \quad p^*_{s2} < p_t \le \bar{p} \\ \underline{\mathrm{P}}_j(p_t) & \text{if} \quad p_t \le p^*_{s2} \end{cases} \tag{52}$$

If instead the senior creditor is restructured first, that is $x_s < x_j$, claim values are given by:

$$S(p_t) = \begin{cases} \frac{b_s}{r} + \left(\underline{P}_j(\bar{p}) + \underline{P}_s(\bar{p}) - J(\bar{p}) - \frac{b_s}{r}\right)\left(\frac{p_t}{\bar{p}}\right)^{\lambda} & \text{if} \quad p_t > \bar{p} \\ \underline{P}_j(p_t) + \underline{P}_s(p_t) - J(p_t) & \text{if} \quad p^*_{j2} < p_t \leq \bar{p} \\ \underline{P}_s(p_t) & \text{if} \quad p_t \leq p^*_{j2} \end{cases} \tag{53}$$

$$J(p_t) = \begin{cases} \frac{b_j}{r} + \left(\underline{P}_j(p^*_{j2}) - \frac{b_j}{r}\right)\left(\frac{p_t}{p^*_{j2}}\right)^{\lambda} & \text{if} \quad p_t > p^*_{j2} \\ \underline{P}_j(p_t) & \text{if} \quad p_t \leq p^*_{j2} \end{cases} \tag{54}$$

where

$$\underline{P}_e = \xi_e(V - V_L) \tag{55}$$
$$\underline{P}_j = \xi_j(V - V_L) \tag{56}$$
$$\underline{P}_s = \xi_s(V - V_L) + V_L. \tag{57}$$

If $x_s = x_j$, all debt classes are restructured at once and debt-holders guarantee their Nash bargaining outcome $\underline{P}_s$ and $\underline{P}_j$. Figures 2 shows the equilibrium claim values resulting from three alternative scenarios where the senior creditor is impaired first (in Panel A), jointly (Panel B) or after the junior has been impaired (Panel C).

Given the creditors' bargaining power $\xi_s + \xi_j$ and the face value F, the priority structure of claims does not affect the equity value. The debt value, $S(p_t) + J(p_t)$, remains the same regardless of the allocation of face value among creditors. This result is consistent with MBP (1997) in that the equity value is affected by the overall face value and the bargaining power of the equity holder. Moreover, it extends MBP results to a multiple creditors scenario, in that the allocation of debt among classes is irrelevant.

Furthermore, by preventing inefficient liquidation, strategic debt service eliminates direct bankruptcy costs. This result is consistent with the Modigliani-Miller theorem in terms of irrelevance of the debt priority structure.

Further analytical results, directly related to the equilibrium claim values, are provided in Section VI where we investigate the implication of sequential restructuring on the credit spreads.

C) Restructuring plan. The equity holders file either a restructuring plan $\mathcal{P}_j$ or a plan $\mathcal{P}_s$, depending on the level of our parameters. After impairing either class of claims they will propose an equilibrium plan $\mathcal{P}_{s,j}$.

The plan $\mathcal{P}_j$ consists of a promise to pay the following coupon flows:

$$\begin{cases} b_j(p_t) = \alpha_{\xi_{s,j}} p_t + (1 - \alpha_{\xi_{s,j}})\gamma r - b_s \\ b_s \end{cases} \Rightarrow \quad \text{for} \quad p_t \leq \bar{p} \tag{58}$$

which impairs the junior creditor while leaving unchanged the contractual coupon to the senior.

When instead the equity holders file a plan $\mathcal{P}_s$ they promise to pay the pair of coupon flows

$$\begin{cases} b_j \\ b_s(p_t) = \alpha_{\xi_{s,j}} p_t + (1 - \alpha_{\xi_{s,j}})\gamma r - b_j \end{cases} \Rightarrow \quad \text{for} \quad p_t \leq \bar{p} \tag{59}$$

which impairs the senior creditor without rescheduling payments for the junior claimant.

After impairing only one creditor by filing a plan $\mathcal{P}_j$ or $\mathcal{P}_s$, the equity holders file a plan $\mathcal{P}_{s,j}$ which consists of a pair of debt service flow functions:

$$\begin{cases} b_j(p_t) = \xi_j(1 - \alpha)(p_t - r\gamma) \\ b_s(p_t) = \alpha_{\xi_s} p_t + (1 - \alpha_{\xi_s})\gamma r \end{cases} \Rightarrow \text{ for } p_t \leq \begin{cases} p^*_{s2} & \text{if } \mathcal{P}_j \text{ has been filed} \\ p^*_{j2} & \text{else} \end{cases} \tag{60}$$

This equilibrium plan impairs both creditors at once.

The three possible pairs of debt service flow functions are shown in Figure 2, where the senior debt is restructured first (Panel A1), jointly (Panel B1) or after the junior has been restructured (Panel C1).

VI. Implications on credit spreads and APV

A. *APV and Renegotiation Premia*

The opportunity to reschedule claims sequentially allows the equity holders to delay renegotiation with the 'strong' creditor until the firm value is sufficiently low and hence the liquidation threat less effective. In principle, seniority strengthens the position of a creditor at the negotiating table. In particular, when bankruptcy costs and firm's scrapping value are high. However, when the senior creditor is a tough renegotiator (with high intensity of effective bargaining power), the equity holders do not restructure her claim and renegotiate vis a vis the junior creditor. That is, if a claim seniority is a potentially valuable asset, in practice it becomes not enforceable at the first (highest) bankruptcy threshold. Therefore, sequential restructuring may aggravate APV compared to a restructuring system where senior creditors retain veto power at any stage of the renegotiation. The additional APV may be simply measured by the difference $\eta = Min\{\underline{P}_s, Fs\} - S$ where $\underline{P}_s$ is the claim value were debt classes restructured all at once. Interestingly, some simple algebra can show that the sign of η depends on the order in which claims are restructured. We highlight this result in the following proposition.

Proposition 5. *If the senior claim is not restructured first, the difference $\eta(p_t)$ is positive for $p_t \in (p_{s2}, \bar{p}]$ and sequential restructuring aggravate APV. If instead the senior claim is restructured first, then $\eta(p_t)$ is negative for $p_t \leq \bar{p}$ and sequential restructuring reduces APV. When all classes are restructured jointly: $\eta(p_t) = 0$ for $p_t \leq \bar{p}$.*

Proof In the private game, an impaired creditor accepts a plan which guarantees him at least with his Nash bargaining share in the joint game. Therefore, if the junior claim is restructured first, for $p_t \in (p_{s2}, \bar{p}]$, the value of the junior debt is such that $J > \underline{P}_j$. Also, by Proposition 1 $J = \underline{P}_s + \underline{P}_j - S$. Therefore, $\underline{P}_s + \underline{P}_j - S > \underline{P}_j$, that is, $\underline{P}_s - S > 0$ and $\underline{P}_j - J < 0$. If instead the senior claim is restructured first, by a symmetric reasoning, it will be $\underline{P}_s - S < 0$ and $\underline{P}_j - J > 0$. ■

Proposition 5 confirms our previous intuition: when senior classes are left unimpaired, seniority is not enforceable. Therefore, senior creditor with higher intensity of effective bargaining power suffers stronger APV under Chapter 11.

The impossibility of enforcing seniority and the resulting additional APV has substantial impact on renegotiation premia and hence on the spreads. The percentage contribution of sequential renegotiation on the senior credit spread can be measured as

$$rp_s = \frac{Min\{V_L, F_s\} - S}{F_s - S} \tag{61}$$

During restructuring, if a creditor can exercise his outside option via liquidation, one should expect the renegotiation premium to be negative. That is, the possibility of rescheduling debt should reduce the spreads during restructuring. In sequential renegotiation, this conclusion still holds with respect to junior creditors. Regardless of the impairment strategy, the renegotiation premium to the junior creditor is always negative[27]. Unlike junior creditors, if senior claimholders are left unimpaired, by losing their veto power they may lose a valuable outside option. Therefore, depending on the impairment strategy, the renegotiation premium of senior creditors may be positive or negative.

Rewriting the renegotiation premium as

$$rp_s = \frac{Min\{V_L, F_s\} - \underline{P}_s}{F_s - S} + \frac{\underline{P}_s - S}{F_s - S} \tag{62}$$

helps at identifying which factors determine the sign of rp_s. The term $\frac{Min\{V_L,F_s\}-\underline{P}_s}{F_s-S}$ is always negative and it captures the loss of value[28] from liquidating the firm instead of restructuring all classes at once. We will refer to this term as to *percentage efficiency loss.* The second term, $\frac{\underline{P}_s-S}{F_s-S}$, as argued in Proposition 5, can be positive or negative depending on the impairment strategy. When positive, this term captures the additional APV[29] as discussed earlier. Therefore, the renegotiation premium can be positive only if the senior creditor is not impaired at $p_t = \bar{p}$ and therefore the additional APV more than offsets the benefits from collective restructuring.

Our results are summarized in Figure 3, which shows the renegotiation premium rp_s evaluated at $\bar{p}$ and its two components (percentage efficiency loss and additional APV) as depending on b_s (which varies from 0 to 0.2 with total coupon payment, $b_s + b_j$, constant and equal to 0.2). Note that, once negative, the renegotiation premium does not continue decreasing when b_s increases. This is because the contribution of the efficiency loss at reducing the spreads decreases as more face value is allocated to the senior creditor[30]. Even though the contribution of the efficiency loss becomes smaller with F_s, once F_s is sufficiently high (such that $\frac{\alpha\xi_s}{F_s-\gamma} \leq \frac{\xi_j(1-\alpha)}{F_j}$) the impairment strategy reverts and the senior creditor is impaired first. From this level of F_s, the additional APV becomes negative, that is sequential renegotiation reduces APV, and the renegotiation premium will tend to decrease again[31].

A.1. *Positive Renegotiation Premia: some empirical findings and legal issues.* Our argument that Chapter 11 may aggravate APV when senior classes are unimpaired can be of great relevance in practice. It is quite common that creditors on top of the priority line (senior secured creditors) are not impaired by a plan and when they are, the impairment involves all classes (see LoPucki (2004) and LoPucki (1993)). In other words, the impairment structure of a plan is often in line with the priority structure of claims.

Also, our analysis is consistent with the fact that at times secured unimpaired creditors undertake legal actions to exit their *powerless position* of non-voting class. If a claim is 'actually unimpaired', in the sense that there is no loss of value after restructuring, it may be difficult to justify costly litigation[32]. In a Chapter 11 proceeding unimpaired creditors often file a number of objections to confirmation of a plan and/or motions to convert to Chapter 7 or to lift (debtor) from automatic stay. Motions and objections to confirmation typically address: improper classification and treatment of classes[33] and lack of feasibility of the plan[34]. Even though objections by unimpaired creditors are quite common, courts typically hold that a creditor whose rights are unimpaired under the plan has no right to object to confirmation (see In re Wonder Corp. of America, Bankruptcy Court, District of Connecticut, case n.70 B.R. 1018, 1023, 1987).

Even though there may be no legal impairment, the default risk of senior/secured debt arises due to the possibility of further deterioration of the assets value and the need of restructuring senior/secured classes too. According to the Code, a plan must be feasible in the sense of not likely to be followed by liquidation or need of further financial reorganization (Bankruptcy Code, Section 1129, Paragraph (a)(11)). However, as repeatedly held by Bankruptcy Courts, the concept of feasibility simply involves the reasonable prospects of financial stability and success. It is not necessary that success be guaranteed, but only that there may be a reasonable expectation of success. The mere prospect of financial uncertainty cannot defeat confirmation on feasibility grounds since a guarantee of the future is not required (see In re Drexel Burnham

Lambert Group Inc. Bankruptcy Court, Southern District of New York, case n. 138 B.R. 723, 1992). Most important, empirical evidence shows that a large number of firms restructure their debt a second time. Using a sample of 197 public companies over the period 1979 − 88, Hotchkiss (1995) finds that 32% of firms restructure their debt a second time either in Chapter 11 or in a private workout. Similarly, according to Gilson (1997), almost 25 percent of firms file for bankruptcy or restructure their debts a second time.

It is worth to note that the possibility that an unimpaired senior/secured creditor receives a value, S, below his collateral, V_L, is not ruled out by the Bankruptcy Code. In particular, the Code restricts the application of what is known as "best interest test" to *impaired creditors*[35] (this test requires that impaired creditors must receive at least what they would receive in a Chapter 7 liquidation). Therefore, as the best interest test does not apply to unimpaired classes[36] nothing prevents positive renegotiation premia in Chapter 11.

The idea that Chapter 11 rules are particularly disadvantageous to secured creditors is well recognized by legal scholars. Bebchuck and Fried (1996) argue that Chapter 11 rules tend to redistribute value from secured to unsecured creditors and to equity holders and that secured creditors may receive less than what they would receive in a Chapter 7 liquidation. Also, the general idea of absolute priority violation in Chapter 11 is supported by several empirical studies suggesting that junior creditors and equity holders receive non-zero distribution before secured and/or senior creditors are fully paid. Violation of APR in Chapter 11 reorganizations has been documented by Weiss (1990), Eberhart et al. (1990), Altman (1991), Fabozzi et al. (1993), Altman and Eberhart (1994), Franks and Torous (1994). In particular, the possibility of positive renegotiation premia is in line with Pulvino and Pidot (1997) who find that bonds with very high collateral ratios (which, they argue, in principle, should be immune to default risk) yield 160 basis points above highly-rated bond yields[37].

B. *Spread reversals between senior and junior spread*

Finally, another interesting implication of our model emerges when comparing the credit spreads of the senior and the junior creditors, which are calculated as $CS_s = \frac{b_s}{S} - r$ and $CS_j = \frac{b_j}{J} - r$ respectively. In Figure 4 and Figure 5 we show the two spreads, as functions of b_s. In particular, using the same parameters as in Figure 3, in Figure 4 the spreads are evaluated at different level of p_t (with p_t above, equal and below the restructuring threshold). With same baseline parameters, in Figure 5, we show the spreads, still as functions of b_s, but for three different level of total face value (with b equal to $b1 = 0.25$, $b2 = 0.30$ and $b3 = 0.35$) and evaluated at the same level of p_t. In both figures, for high enough level of senior face value there is reversal between senior and junior spreads.

Interestingly, we are able to derive a simple sufficient condition for avoiding reversals for all levels of the state variable. We do this by comparing the junior spread and the spread of the senior claim stripped for the scrapping value γ (shortly referred to as senior 'unsecured'claim).

We start decomposing the senior spread to isolate the effect of the secured part of the claim. The spread of the senior claim can be arranged as follows

$$\begin{aligned} CS_s &= r\frac{F_s - S}{S} \\ &= r\frac{(F_s-\gamma)-(S-\gamma)}{S}\cdot\frac{S-\gamma}{S-\gamma} \\ &= r\frac{\hat{F}_s - \hat{S}}{\hat{S}}\cdot\frac{\hat{S}}{S} \end{aligned} \tag{63}$$

where $\hat{S} = S - \gamma$ and $\hat{F}_s = F_s - \gamma$. The two factors in equation (63) can be interpreted as follows. The first term, denoted as

$$\widehat{CS}_s = r\frac{\hat{F}_s - \hat{S}}{\hat{S}}, \tag{64}$$

represents the credit spread on the senior 'unsecured' claim.

The second term, $\hat{S}/S$, captures the effect of the 'fully-secured' part of the claim on the overall credit spread CS_s ('fully-secured' is used to mean that S can never fall below γ). In fact, the bigger the fully-secured part of the claim the smaller the ratio $\hat{S}/S$ and the spread CS_s.

We can now compare the spreads of the two 'unsecured' claims by calculating the difference $\widehat{CS}_s - CS_j$, which by some simple algebra yields

$$\widehat{CS}_s - CS_j = r\frac{\hat{F}_s J - \hat{S}F_j}{\hat{S}J}. \tag{65}$$

Proposition 6. *The difference $\widehat{CS}_s - CS_j$ is positive if and only if the equilibrium impairment strategy is $\{p^*_{j2}, \bar{p}\}$. $\widehat{CS}_s - CS_j$ is negative if and only if the impairment strategy is $\{p^*_{s2}, \bar{p}\}$. The equality holds, and $\widehat{CS}_s - CS_j = 0$ when $p^*_{j2} = p^*_{s2} = \bar{p}$.*

Proof See Appendix. ■

Proposition 6 provides a sufficient condition for detecting the case where $CS_s < CS_j$ for any level of the state variable. In fact, the inequality $CS_s < CS_j$ rearranges into $\widehat{CS}_s\hat{S}/S < CS_j$, which holds if $\widehat{CS}_s < CS_j$ because the term $\hat{S}/S$ is always smaller than one.

Last, when the collateral γ tends to zero, our sufficient condition becomes also necessary condition (because, by definition of $\widehat{CS}_s$, with $\gamma = 0$ then $\widehat{CS}_s = CS_s$). This immediate result can be summarized in the following proposition.

Proposition 7. *If γ tends to zero, the difference $CS_s - CS_j$ is positive if and only if the impairment strategy is $\{p^*_{j2}, \bar{p}\}$. $CS_s - CS_j$ is negative if and only if the impairment strategy is $\{p^*_{s2}, \bar{p}\}$. When $p^*_{j2} = p^*_{s2} = \bar{p}$, then $CS_s - CS_j = 0$.*

In general, low collateral value V_L (that is, low α and/or γ) makes reversal of the spreads more likely because the effective bargaining strength of the senior creditor decreases and that of the junior increases (with α and/or γ decreasing). The argument runs symmetric to what argued in Section A. When a senior creditor is impaired first, seniority is enforceable, however there is little value attached to it because the liquidation threat has little scope.

In particular, we are able to identify two circumstances in which the possibility of reversal is of some practical relevance.

First, in large reorganizations involving more complex priority structures (with at least four or more classes of debt, such as senior secured, senior unsecured, senior subordinated and junior), it is often the case that low priority classes, such as senior unsecured (or subordinated) and junior classes, receive very similar distribution, and seniority provisions do not necessarily add value to a claim. According to Altman and Eberhart (1994), unlike high priority classes, lower priority senior-subordinated and subordinated bonds perform very similarly and do very poorly at emergence from Chapter 11. They find an average loss on emergence from Chapter 11 (during the period $1980-92$) of 76.4% and 87.4% for senior subordinated and subordinated bonds respectively. Franks and Torous (1994) find that senior unsecured and junior creditors recover respectively 47% and 28.9% in formal bankruptcy, however in informal restructuring the recovery increases and reverts to 78.5% for senior unsecured and 79.6% for junior debt[38]. It is beyond the scope of this paper to extend the model to account for more complex debt structures. However, the empirical finding on recovery in Chapter 11 is in line with our predictions because classes at the bottom of the priority hierarchy may be left with very little collateral so observing similar or inverted recovery rates between senior and junior spreads might be more likely.

Second, when the senior creditor is secured as in our setting, for reasonable parameters (as in Figure 3 where, at the threshold level $\bar{p}$ the bankruptcy costs, $(V - V_L)/V$, are just below 25%), then reversal can occur only for a particularly high level of F_s. That is, the reversal seems to be essentially a *reversal between very large claims versus very small unsecured claims.* Reversal would be consistent with the treatment of *convenience claims* (small unsecured claims, which for administrative convenience are placed in a separate class) which in Chapter 11 reorganizations are often unimpaired.

Last, one should note that the possibility of spread reversals is not a structural feature of Chapter 11. Actually, *Chapter 11 may reduces the scope of reversals.* If the senior creditor is impaired first, from Proposition 5 (see proof) there is a redistribution of value from the junior creditor (the strong player in this case) to the senior creditor. That is, in this case, compared to an all-at-once restructuring system, Chapter 11 reduces APV between senior and junior creditors. Consequently, under a joint-restructuring system reversal might be enhanced because senior spread should be even higher and junior spread smaller.

VII. **Conclusions**

The model discussed above provides a framework for a positive analysis of Chapter 11, which integrates legal and economics features. When restructuring involves multiple creditors, the equity holders strategically enforce the set of bankruptcy rules which determines a unique equilibrium restructuring plan.

The current model provides diverse economic implications. When the equity holders can renegotiate with one creditor at a time, as in Chapter 11, the strategic decision concerns not only the timing of bankruptcy but also the order in which creditors will be impaired. We find that equity holders impair creditors with a smaller intensity of effective bargaining strength first; that is, the ratio of effective bargaining power to unsecured face value. Depending on the value of the above ratio, in equilibrium the equity holders will strategically default on a single class of claims when the state variable reaches a certain trigger level. If the firm cash flows and the asset value continue to decrease to a second (lower) trigger level, then the equity holders will jointly default on both claims.

We find that for reasonable parameter values (not excessively high bankruptcy costs and senior face value relative to junior face value), an equilibrium plan impairs junior classes and leaves senior classes unimpaired at the first (higher) bankruptcy threshold. Senior claimholders are restructured jointly with junior creditors only if cash flows fall below a lower threshold level. Interestingly, this kind of equilibrium is consistent with Chapter 11 plans, which commonly leave senior/secured creditors unimpaired. In this scenario, when senior claimholders are left unimpaired, by losing their veto power they cannot enforce their seniority. In turn, this furthers APV and, in particular, the extent of additional APV may lead to positive renegotiation premia (that is, the senior debt value may fall below its collateral value). The renegotiation premium increases up to 100% when the collateral (firm liquidation value) increases. In other words, at the higher bankruptcy threshold level, even if senior secured creditors should be in principle immune to default risk because their collateral is high, in practice, as unimpaired class, they cannot enforce liquidation and seize their collateral. As this scenario is the most common in reality, a strong violation of the absolute priority rule seems to be a structural feature of Chapter 11, as well as non-zero default risk in highly secured senior debt.

Moreover, the senior credit spread is not necessarily smaller than the junior one for any level of the state variable. Whether the spreads may revert or not depends on the impairment strategy. In general, low collateral value and high senior face value (relative to junior claims) make reversal of the spreads more likely because the senior creditor's effective bargaining strength may be smaller than the junior one. In particular, an equilibrium plan, which impairs the junior creditor first, guarantees that the senior spreads is below the junior one for any level of the state variable.

Our argument that reversal is more likely to occur when the firm liquidation value is small is consistent with empirical evidence showing that senior unsecured creditors do not necessarily recover higher distribution than junior creditors.

Also, because reversal is more likely to occur if junior unsecured face value is particularly low (compared to senior secured face value), our model is consistent with the treatment convenience claims, which in Chapter 11 reorganizations are often unimpaired.

Last, single-debt-class restructuring in our model is consistent with the fact that private workouts are often targeted to single classes of claims, and that cross default provisions do not prevent a private renegotiation.

Appendix

Proof $\Pi \geq 0$ *under an equity holders' proposal.*

If the equity holders impair the junior creditor, the equity holders' plan belongs to the set $\mathcal{P}_j = \{P_e, P_j, P_s : P_e \geq \underline{\mathrm{P}}_e, P_j \geq \underline{\mathrm{P}}_j, P_s < \underline{\mathrm{P}}_s\}$. Therefore

$$P_e \geq \underline{\mathrm{P}}_e,$$

can be rearranged (by adding and subtracting V) into

$$V - \underline{\mathrm{P}}_e - (V - P_e) \geq 0.$$

Replacing $V - P_e$ and $V - \underline{\mathrm{P}}_e$ with $P_j + P_s$ and $\underline{\mathrm{P}}_j + \underline{\mathrm{P}}_s$ gives

$$\underline{\mathrm{P}}_j + \underline{\mathrm{P}}_s - P_j - P_s \geq 0.$$

As $P_j = J$ and $P_s = S$ the above rewrites as $\underline{\mathrm{P}}_j + \underline{\mathrm{P}}_s - J - S \geq 0$, which is the definition of the arbitrage payoff, Π, therefore one concludes $\Pi \geq 0$. The same conclusion holds if the equity holders impairs the senior creditor.

Proof Hypothesis 2 holds.

When the optimal strategy is $\{p^*_{j1}, p^*_{s2}\}$, by substituting for p^*_{s2} and $\underline{p}$, the inequality $F_s > V_L(p^*_{s2})$ rearranges into:

$$\frac{F_s - (1-\alpha)\gamma}{\alpha} > \frac{\lambda}{\lambda - 1}\frac{F_s - (1-\alpha_{\xi_s})\gamma}{\alpha_{\xi_s}} + \frac{\gamma}{1-\lambda}\left(\frac{p^*_{s2}}{\underline{p}}\right)^{\lambda} \tag{66}$$

First, note that the left hand side, $\frac{F_s-(1-\alpha)\gamma}{\alpha}$, is greater than $\frac{F_s-(1-\alpha_{\xi_s})\gamma}{\alpha_{\xi_s}}$ (because $\alpha_{\xi_s} > \alpha$) and in turn, one can find that

$$\frac{F_s - (1-\alpha_{\xi_s})\gamma}{\alpha_{\xi_s}} > \frac{\lambda}{\lambda - 1}\frac{F_s - (1-\alpha_{\xi_s})\gamma}{\alpha_{\xi_s}} + \frac{\gamma}{1-\lambda}\left(\frac{p^*_{s2}}{\underline{p}}\right)^{\lambda}$$

holds because it rearranges into $\frac{p^*_{s2}}{\underline{p}} > \left(\frac{p^*_{s2}}{\underline{p}}\right)^{\lambda}$ (which holds being $p^*_{s2}/\underline{p} > 1$ and $\lambda < 0$).

(By following the same calculation, just replacing the subscript s with c and p^*_{s2} with p^*_c, one can prove that Hypothesis 2A is satisfied, that is: $F_c > V_L(p^*_c)$.)

Similarly, when the optimal strategy is $\{p^*_{j2}, p^*_{s1}\}$, the inequality $F_s > V_L(p^*_{s1})$ rearranges into:

$$\frac{F_s-(1-\alpha)\gamma}{\alpha} > \frac{\lambda}{\lambda-1}\frac{F-(1-\alpha_{\xi_{s,j}})\gamma}{\alpha_{\xi_{s,j}}} + \frac{\gamma}{1-\lambda}\left(\frac{p^*_{s1}}{\underline{p}}\right)^{\lambda} \tag{67}$$

which holds because, as shown below, the left hand side of equation (67), $\frac{F_s-(1-\alpha)\gamma}{\alpha}$, is greater than $\frac{F_s-(1-\alpha_{\xi_{s,j}})\gamma}{\alpha_{\xi_{s,j}}}$ and in turn one can rearrange

$$\frac{F_s-(1-\alpha_{\xi_{s,j}})\gamma}{\alpha_{\xi_{s,j}}} > \frac{\lambda}{\lambda-1}\frac{F-(1-\alpha_{\xi_{s,j}})\gamma}{\alpha_{\xi_{s,j}}} + \frac{\gamma}{1-\lambda}\left(\frac{p^*_{s1}}{\underline{p}}\right)^{\lambda}$$

into $\frac{p^*_{s1}}{\underline{p}} > \left(\frac{p^*_{s1}}{\underline{p}}\right)^{\lambda}$ (which holds because $p^*_{s2}/\underline{p} > 1$ and $\lambda < 0$). Last, as we claimed,

$$\frac{F_s-(1-\alpha)\gamma}{\alpha} > \frac{F_s-(1-\alpha_{\xi_{s,j}})\gamma}{\alpha_{\xi_{s,j}}}$$

because it rearranges as $F-\gamma > \frac{F_j\alpha_{\xi_{s,j}}}{(\xi_s+\xi_j)(1-\alpha)}$ which, in turn, holds because by Proposition 2 p^*_{s1} is an optimal restructuring threshold if and only if $F-\gamma > F_j\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)}$.

Proof Propositions 3 and 4.

As argued in Section III, to rule out arbitrage opportunities, the claim values $S(p_t)$ and $J(p_t)$ must satisfy the following equations:

$$rS(p_t) = b_s(p_t) + \mu p_t S'(p_t) + \frac{\sigma^2}{2}{p_t}^2 S''(p_t), \tag{68}$$

$$rJ(p_t) = b_j(p_t) + \mu p_t J'(p_t) + \frac{\sigma^2}{2}{p_t}^2 J''(p_t). \tag{69}$$

Proof Proposition 3. When the optimal strategy is $\{p^*_{s2}, p^*_{j1}\}$, first, one can derive the service flow function $b_s(p_t)$ as follows. For $p_t \le p^*_{s2}$, by equation 27, the senior value is $S = \underline{P}_s$. Smooth-pasting conditions are satisfied because p_{s2} minimizes S (and $\partial S/\partial p_{s2} = 0$ implies $\partial S(p_t)/\partial p_t \mid_{p_{s2}} = \partial \underline{\mathrm{P}}_s(p_t)/\partial p_t \mid_{p_{s2}}$). Substituting for S, S' and S'' in equation (68) yields

$$b_s(p_t) = \alpha_{\xi_s} p_t + (1-\alpha_{\xi_s})\gamma r \quad \text{for} \quad p_t \le p^*_{s2}$$

Similarly we can derive $b_j(p_t)$. For $p_t \le p^*_{j1}$, by equation (30), the junior value is $J = (\underline{P}_s + \underline{P}_j - S)$. Smooth-pasting conditions are satisfied because: i) p_{j1} minimizes J (and this implies continuity in first derivative at p_{j1}) and ii) at p_{s2} continuity of S in the first derivative also guarantees continuity of J in the first derivative (i.e. $\partial J(p_t)/\partial p_t \mid_{p_{s2}} = \partial(\underline{\mathrm{P}}_s(p_t) + \underline{\mathrm{P}}_j(p_t) - S(p_t))/\partial p_t \mid_{p_{s2}} = \partial \underline{\mathrm{P}}_j(p_t)/\partial p_t \mid_{p_{s2}}$). Substituting for J, J' and J'' in equation (69) delivers

$$b_j(p_t) = \alpha_{\xi_{s,j}} p_t + (1-\alpha_{\xi_{s,j}})\gamma r - b_s(p_t) \quad \text{for} \quad p_t \le p^*_{j1}$$

Proof Proposition 4. The proof runs symmetric to the previous one. When the optimal strategy is $\{p^*_{s1}, p^*_{j2}\}$, first, we derive the service flow function $b_j(p_t)$. For $p_t \le p^*_{j2}$, by equation (33), the junior value is $J = \underline{P}_j$. Smooth-pasting conditions are satisfied because p^*_{j2} minimizes J. Substituting for J, J' and J'' in equation (69) yields

$$b_j(p_t) = \xi_j(1-\alpha)(p_t - \gamma r) \quad \text{for} \quad p_t \le p^*_{j2}$$

Then one can derive $b_s(p_t)$ as follows. For $p_t \le p^*_{s1}$, by equation (35), the senior value is $S = (\underline{P}_s + \underline{P}_j - J)$. The function S satisfies smooth-pasting conditions because i) p_{s1} minimizes S at p_{s1} and ii) continuity of J in the first derivative at p_{j2} guarantees continuity of S in first derivative. Substituting for S, S' and S'' in equation (69) gives

$$b_s(p_t) = \alpha_{\xi_{s,j}} p_t + (1 - \alpha_{\xi_{s,j}})\gamma r - b_j(p_t) \quad \text{for} \quad p_t \leq p^*_{s1}$$

Last, note that under both strategies, S and J satisfy the no-bubble conditions: $\underline{\mathrm{P}}_s(\underline{p}) = \gamma$, $\underline{\mathrm{P}}_j(\underline{p}) = 0$ and $\lim_{p_t\to\infty} S(p_t) = \frac{b_s}{r}$, $\lim_{p_t\to\infty} J(p_t) = \frac{b_j}{r}$.

Proof Proposition 6.

Notice first that the sign of $\widehat{CS}_s - CS_j$ is positive if and only if the numerator is positive; that is

$$\frac{(b_s/r - \gamma)}{b_j/r} > \frac{(S-\gamma)}{J}. \tag{70}$$

Notice that regardless of the optimal stopping strategy, for low enough level of the state variable the equity holders impair both creditors jointly. And, when joint renegotiation occurs (i.e., $p_t \in [\underline{\mathrm{p}}, p^*_{i2}]$) the value of the junior and senior debt is given by

$$\begin{aligned} J = \underline{\mathrm{P}}_j &= \xi_j(V - V_L) \\ S = \underline{\mathrm{P}}_s &= \xi_s(V - V_L) + V_L. \end{aligned}$$

Substituting for the debt values, $\underline{\mathrm{P}}_j$ and $\underline{\mathrm{P}}_s$, when $p_t \in [\underline{\mathrm{p}}, p^*_{i2}]$, and replacing F_s with $F - F_j$, inequality (70) rearranges as

$$\frac{b_j}{r}\frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} < F - \gamma.$$

We have already found this inequality in comparing p^*_{i2} and $\bar{p}$ and we know that it holds if the optimal stopping strategy is $\{p^*_{j2}, \bar{p}\}$ (see Proposition 1). The reverse inequality holds instead if the optimal stopping strategy is $\{p^*_{s2}, \bar{p}\}$. Therefore we conclude that

$$\text{for } p_t \in [\underline{\mathrm{p}}, p^*_{i2}] \begin{cases} \widehat{CS}_s - CS_j > 0 \ \text{ iff } \ \frac{(b_s/r-\gamma)}{b_j/r} > \frac{(S-\gamma)}{J} \ \text{ iff } \ \{p^*_{i2} = p^*_{j2}\} \\ \widehat{CS}_s - CS_j = 0 \ \text{ iff } \ \frac{(b_s/r-\gamma)}{b_j/r} = \frac{(S-\gamma)}{J} \ \text{ iff } \ \{p^*_{i2} = \bar{p}\} \\ \widehat{CS}_s - CS_j < 0 \ \text{ iff } \ \frac{(b_s/r-\gamma)}{b_j/r} < \frac{(S-\gamma)}{J} \ \text{ iff } \ \{p^*_{i2} = p^*_{s2}\} \end{cases} \tag{71}$$

Left to prove is that the sign of the spread doesn't revert for $p_t \in (p^*_{i2}, \bar{p}]$ (we prove this below, see point (i)) and $p_t \in (\bar{p}, \infty)$ (see point (ii)).

(i) We prove here that the sign of $\widehat{CS}_s - CS_j$ does not revert when $p_t \in (p^*_{i2}, \bar{p}]$. To do this let us compare our firm with an identical one, say firm A, which only differs for allocation of bargaining power amongst creditors, in particular:

$$\xi_j > \xi_{Aj} \ \text{ with } \ \xi_s + \xi_j = \xi_{As} + \xi_{Aj},$$

(where the subscript A denotes the creditors' bargaining powers in firm A).

Also, assume that ξ_j and ξ_{Aj} are such that:

$$\xi_{Aj}: \quad \frac{\alpha_{\xi_{s,j}}}{\xi_{Aj}(1-\alpha)} = \frac{F-\gamma}{b_j/r} \quad \text{and} \quad \xi_j: \quad \frac{\alpha_{\xi_{s,j}}}{\xi_j(1-\alpha)} < \frac{F-\gamma}{b_j/r}.$$

which implies (by Proposition 2) that the optimal stopping strategy is to impair both creditors at $\bar{p}$ in firm A and the senior creditor first in the other firm.

From our assumptions on $\{\xi_{Aj}, \xi_j\}$ and our previous result in equation (71), it follows that

$$\text{for } \ p_t \in [\underline{\mathrm{p}}, \bar{p}] \to \frac{b_s/r - \gamma}{b_j} = \frac{\underline{\mathrm{P}}_{As} - \gamma}{\underline{\mathrm{P}}_{Aj}} \tag{72}$$

$$\text{for } \ p_t \in [\underline{\mathrm{p}}, p^*_{j2}] \to \frac{b_s/r - \gamma}{b_j} > \frac{\underline{\mathrm{P}}_s - \gamma}{\underline{\mathrm{P}}_j}, \tag{73}$$

where $\underline{\mathrm{P}}_{As}$ and $\underline{\mathrm{P}}_{Aj}$ correspond to the functions $\underline{\mathrm{P}}_s$ and $\underline{\mathrm{P}}_j$ with bargaining powers ξ_{As} and ξ_{Aj} respectively.

By comparing equations (72) and (73), one can easily find that

$$\text{for } p_t \in (p_{j2}^*, \bar{p}] \rightarrow \frac{\underline{\mathrm{P}}_{As} - \gamma}{\underline{\mathrm{P}}_{Aj}} = \frac{b_s/r - \gamma}{b_j} > \frac{S - \gamma}{J}, \tag{74}$$

that is, the sign of the spread does not revert. This can be done by rearranging equation (74) as $(\underline{\mathrm{P}}_{As} + \underline{\mathrm{P}}_{Aj} - \gamma)/\underline{\mathrm{P}}_{Aj} > (S + J - \gamma)/J$ and noting that: $\underline{\mathrm{P}}_{As} + \underline{\mathrm{P}}_{Aj} = \underline{\mathrm{P}}_s + \underline{\mathrm{P}}_j$ (because $\xi_s + \xi_j = \xi_{As} + \xi_{Aj}$ by assumption) and $\underline{\mathrm{P}}_s + \underline{\mathrm{P}}_j = S + J$ (by Proposition 1). It follows that inequality (74) simplifies into $J > \underline{\mathrm{P}}_{Aj}$, which holds because we have assumed that $\xi_j > \xi_{Aj}$.

When the optimal stopping strategy is $\{p_{s2}^*, \bar{p}\}$, one can symmetrically repeat the above argument. Just compare a firm, which has a stopping strategy $\{p_{s2}^*, \bar{p}\}$, with the same benchmark firm A and $\xi_j < \xi_{Aj}$. The rest of the proof runs symmetrically, and can be done by simply reverting the sign of the inequalities.

(ii) From some straightforward algebra it is immediate to see that the sign of the spread cannot change when $p_t \in (\bar{p}, \infty)$. We know that the sign of the spread is determined by the following inequality

$$\frac{(b_s/r - \gamma)}{b_j/r} \gtreqqless \frac{(S - \gamma)}{J}.$$

When the stopping strategy is $\{p_{j2}^*, \bar{p}\}$, substituting for the values S and J and rearranging yields

$$\left(\underline{P}_j(p_{j2}^*) - \frac{b_j}{r}\right) \bar{p}^{\lambda} \gtreqqless \left(\underline{P}_s(\bar{p}) + \underline{P}_j(\bar{p}) - J(\bar{p}) - \frac{b_s}{r}\right) {p_{j2}^*}^{\lambda},$$

which is independent of p_t. Adding to this result the fact that the difference of the spread is continuous at $p_t = \bar{p}$, we conclude that $\widehat{CS}_s - CS_j$ cannot revert sign in the range $p_t \in (\bar{p}, \infty)$.

We reach the same conclusion when the stopping strategy is $\{p_{s2}^*, \bar{p}\}$, by a similar calculation one can easily prove that the sign of the spread is independent of the state variable.

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Notes

[1]A creditor is said to be unimpaired if the plan calls for no scaling down of the coupon payment scheduled in the existing contract.

[2]Companies emerging from bankruptcy often re-enter Chapter 11 within few years. According to Gilson (1997), almost 25 percent of firms file for bankruptcy or restructure their debts a second time. See also Hotchkiss, (1995).

[3]Yet, the 2002 UK Enterprise Act seems to have reduced the power of senior creditors in the event of default.

[4]This is a standard assumption in cash-flow models for corporate debt valuation. Among others, see Mello and Parsons (1992), Fries, Miller and Perraudin (1997), Mella-Barral and Perraudin (1997), Mella-Barral (1999).

[5]Risk-aversion can be easily accounted for by using risk-adjusted probabilities as shown by Harrison and Kreps (1979).

[6]We do not provide the derivation of the firm value through the stochastic calculus. We refer the reader to Dixit and Pindyck (1994) for a general analysis of entry and exit decisions under uncertainty.

[7]The new owner receives the firm value (free of debt) $V(p_t)$ and pays the agreed price $V_L(p_t)$ to the initial owner.

[8]Their formulation is $V_L = \alpha V$ (in our notation).

[9]Otherwise, if $F_s < \gamma$, the senior claim would be fully secured for any level of the state variable and renegotiation would involve only the junior unsecured claim.

[10]See Kordana and Posner (1999) for a critical and exhaustive analysis of Chapter 11's rules.

[11]Chapter 11 allows for involuntary bankruptcy too. However, the Court will dismiss a creditors's bankruptcy petition if the debtor has not failed to pay its debts when due (Bankruptcy Code, Section 303, Paragraph (h)(1)).

[12]Bankruptcy Code, Section 1121.

[13]Bankruptcy Code, Section 1124.

[14]Bankruptcy Code, Section 1126.

[15]Bankruptcy Code, Section 1129, Paragraph (b)(1).

[16]In Chapter 11, the exclusive right lasts 120 days (plus 60 days for securing acceptance of the plan) and can be extended by the Court. The debtors ability to extend the exclusive period has been generally used as leverage in the negotiating process. Under the 2005 Act, the exclusive period cannot be extended beyond a maximum of 18 months (Bankruptcy Code, Section 1129, Paragraph (d)(2)).

[17]For instance, Brown (1989) assumes that only a finite number of proposal is allowed and the bankruptcy court determines the order in which proposals can be voted. Each proposal might rank first, second, or third in the agenda with same probability.

[18]As argued by Welch, a bank, unlike bond-holders, can have a good reputation effect for "tough behavior," which might prevent other borrowers from opportunistic renegotiation.

[19]Fan and Sundaresan use the Nash axiomatic approach with asymmetric bargaining power to model strategic debt service as well as debt/equity swap in a private workout between equity holders and a single class of debt holders. However, apart from the similarity in the Nash bargaining, their study crucially differs from the current model.

[20]See Binmore and Dasgupta (1987) for an extensive analysis of the Nash bargaining solution. We also refer to Fan and Sundaresan (2000) for a similar characterisation of the Nash bargaining solution. For a more detailed derivation of the Nash bargaining solution when uncertainty is modeled through a geometric Brownian motion, see Perraudin and Psillaki (1999).

[21]Equation (17) is derived by applying Ito's lemma to $rC(p_t) = b_c(p_t) + \frac{d}{d\triangle} E_t(C_{t+\triangle}) \mid_{\triangle=0}$.

[22]As argued by Mella-Barral (1999), temporary concessions are much easier to handle compared to irreversible concessions which instead lead to a path-dependent problems.

[23]In fact, if $p_* = min\{p_j, p_s\} = p_s$, then by Hypothesis 2 $V_L(p_s) < F_s$ and therefore, for $p_t < p_s$, players' payoffs are given by equations (22)-(24). If $p_* = min\{p_j, p_s\} = p_j$, i.e. $p_j < p_s$, being $V_L(p_t)$ decreasing in p_t, it holds from Hypothesis 2 that $V_L(p_j) < V_L(p_s) < F_s$; therefore, for $p_t < p_s$, players' payoffs are given by equations (22)-(24).

[24]We denote the optimal trigger by using the subscript 'in', with $i = s, j$ and $n = 1, 2$, to stress the order in which claims are impaired. Therefore, for instance, when the senior creditor is the second claimant to be impaired we denote the optimal trigger level as p_{s2} (while when the senior creditor is impaired first, the trigger level is denoted as p_{s1}).

[25]See equation (15), with $\alpha_{\xi_c} = \alpha_{\xi_{s,j}}$. Also, by Proposition 1 the sum of the (sequentially) renegotiated claim values $S + J$ is equal to $\underline{P}_s + \underline{P}_j$, which is also the total (jointly renegotiated) debt value as argued in Section III.

[26]Because the junior creditor is a residual claimant, she purely benefits from receiving a share of the firm continuation surplus, $V - V_L$, while the senior creditor, due to the priority of his claim, guarantees the full liquidation value V_L.

[27]By Hypothesis 2 (see Appendix), if the junior creditor is left unimpaired, his liquidation payoff at $p_t = \bar{p} = p_{s1}$ is equal to $Max\{(V_L - F_s)^+, 0\} = 0$ (because $V_L(p_{s1}) < F_s$). Therefore, as J is positive, the renegotiation premium to the junior creditor is negative (that is, $rp_j = -J/(F_j - J) < 0$).

[28]By definition of $\underline{\mathrm{P}}_s$ (see equation (6)), $Min\{V_L, F_s\} - \underline{\mathrm{P}}_s$ is equal to $-\xi_s(V - V_L)$.

[29]Note that we have defined the additional APV by using $Max\{\underline{\mathrm{P}}_s, F_s\}$ rather than $\underline{\mathrm{P}}_s$ because, under the absolute priority rule no creditor should receive more than his face value unless more junior claimholders have been fully paid.

[30]While the credit spread increases with F_s increasing, the share of the going concern surplus, $\xi_s(V - V_L)$ (which captures the efficiency loss), does not depend on face values and remains constant.

[31]It goes beyond the scope of this paper, but it can be easily proved that the inflection point of rp_s (as in Figure 3) corresponds to the level of F_s where the two classes are jointly restructured.

[32]We abstract in our model from information problems related to the continuation surplus and liquidation value of the firm. As argued by Bebchuck and Fried (2001), due to uncomplete information on the value of collateral, different (genuine or strategic) estimates of collateral may rise costly and lengthy litigations.

[33]In re: Mirant Corporation, et al. (Bankruptcy Court, Northern District of Texas, case n. 03-46590-DML-11, 2005) the Court denies the motion by a senior creditor (unimpaired and, hence, not entitled to vote), who argues that the plan actually impairs the Senior Notes and so entitled to vote.

[34]In re: Tavern Motor Inn Inc. (Bankruptcy Court, District of Vermont, case n. 56 B.R. 446, 1985), an unimpaired secured creditor filed a motion to convert the case into Chapter 7. The creditor argued the plan (approved by impaired unsecured creditors) actually impairs his class and is *not feasible* because there is a "likelihood of liquidation or further financial reorganization". The motion was denied.

[35]Bankruptcy Code, Section 1129, Paragraph (a)(7).

[36]See Seatco, Inc., (Bankruptcy Court, Northern District of Texas, case n. 00-37332-BJH-11, 2001) where "Class 3 creditors are unimpaired under the Plan and the best interest test is not applicable to them". See also, Bankruptcy Reform Act 1978, House Report (n.95-595) stating "the court may confirm a plan over the objection of a class of secured claims if the members of that class are unimpaired".

[37]Their study uses US Airline secured bond yields and collateral.

[38]They also find that firms entering Chapter 11 have already attempted to restructure informally and Chapter 11 is a last attempt to re-restructure debt when the firm has lower solvency rate.

FIGURES

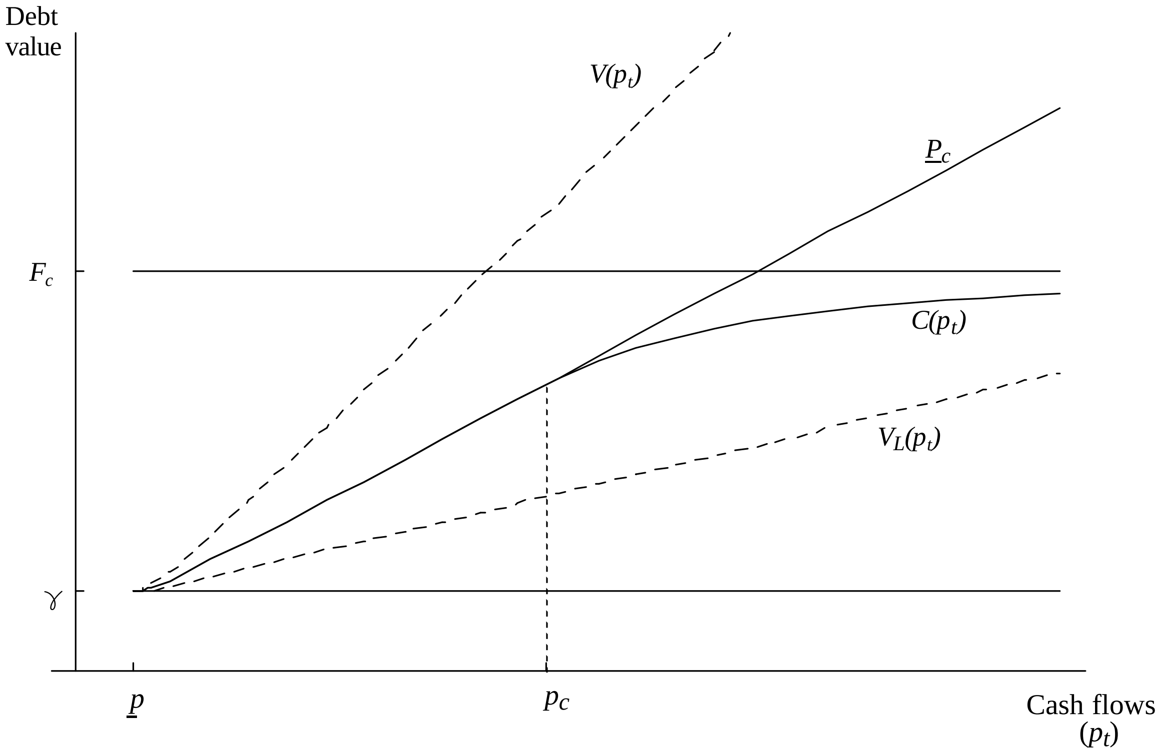

Figure 1. Renegotiated debt value with one single creditor. When renegotiable, the debt value $C(p_t)$ smooth-pastes the Nash bargaining outcome, $\underline{P}_c$, at the threshold level p_c which triggers bankruptcy.

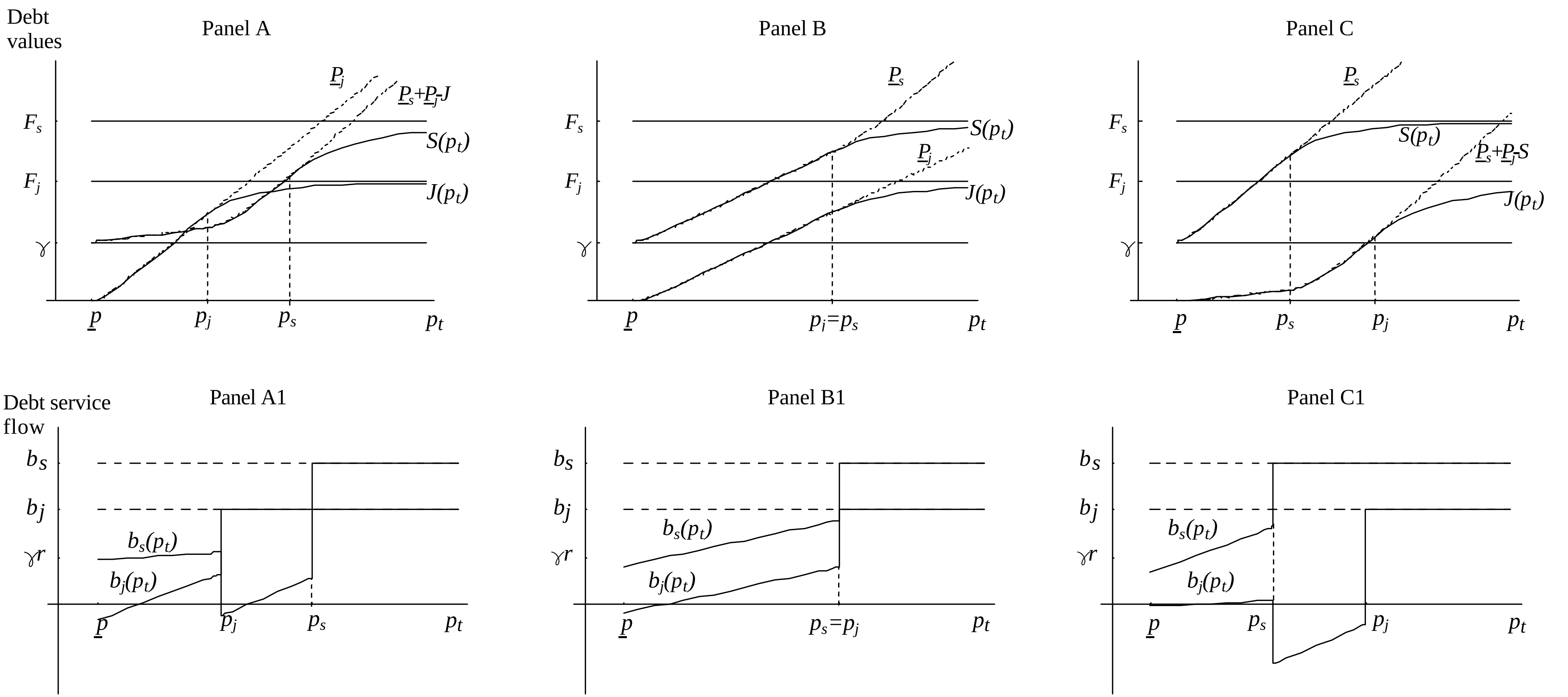

Figure 2. Debt values and service flow functions when $p_s<p_j$, $p_s=p_j$, $p_s>p_j$. In Panel A, the senior debt, $S(p_t)$ is renegotiated first at the trigger level p_s.while the junior debt, $J(p_t)$, is left unimpaired. The senior debt is restructured such that $S(p_t)$ smooth-pastes $\underline{P}_s+\underline{P}_j-J$. The junior debt is restructured when the state variable hits the second lower trigger level p_j where the junior debt smooth-pastes the Nash bargaining share $\underline{P}_j$. In Panel B, all debt classes are collectively restructured. The senior and junior debt values smooth-paste $\underline{P}_s$ and $\underline{P}_j$ respectively and each class guarantees her Nash bargaining outcome. In Panel C, the junior debt, $J(p_t)$ is renegotiated when the state variable hits the threshold level p_j. At this level of p_t, the senior debt, $S(p_t)$, is left unimpaired while the junior debt is restructured so that $J(p_t)$ smooth-pastes $\underline{P}_s+\underline{P}_j-S$. The senior debt is restructured when the state variable hits the second lower trigger level p_s and the senior creditor guarantees his Nash bargaining outcome $\underline{P}_s$. Panels A1, B1 and C1 show the debt service flow functions corresponding to the equilibrium strategy depicted in Panel A, B and C respectively.

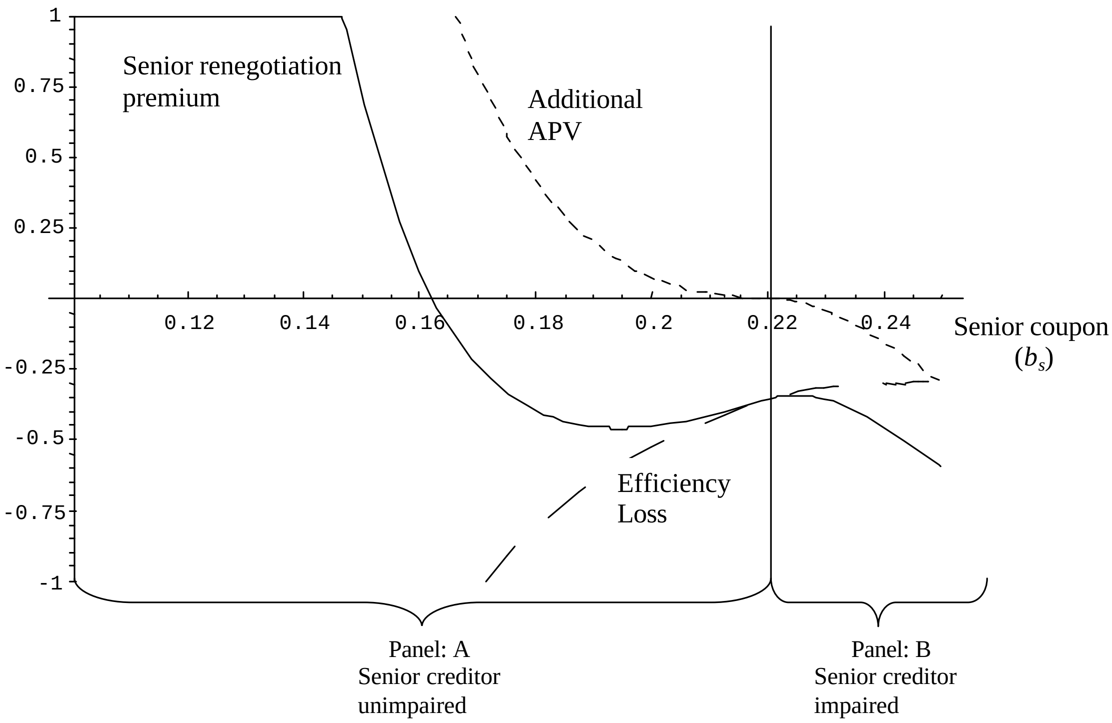

Figure 3. Senior renegotiation premium, APV and Efficiency loss. The renegotiation premium, rp_S, and its components, the additional APV and the efficiency loss, evaluated at $\bar{p}$, are shown as functions of the coupon $b_S \in [0.1, 0.249]$. The total debt face value is held constant with total coupon $b = bj + bs = 0.25$, therefore the bankruptcy threshold level $\bar{p}$ remains constant. When the senior creditor is unimpaired, as in Panel A, the renegotiation premium can be positive or negative. It is positive when the additional APV, always positive in this region, more than offsets the benefits from collective renegotiation measured by the Efficiency loss. If the senior creditor is impaired, as in Panel B, the renegotiation premium is always negative. The vertical line between Panel A and B corresponds to the level of b_S such that both creditors are jointly impaired and hence the additional APV is equal to zero. Baseline parameters are as follows: $r=0.06$, $\mu=0.02$, $s=0.15$, $x_e = x_s = x_j = 1/3$ and $g=1$ (which implies the ratio g/Fs varies from 60% to 24% with b_S increasing from 0.1 to 0.249). The parameter a is chosen such that the loss of value due to inefficient early liquidation, $[V(p_t)-V_L(p_t)]/V(p_t)$, is equal to 25% at $\bar{p}$.

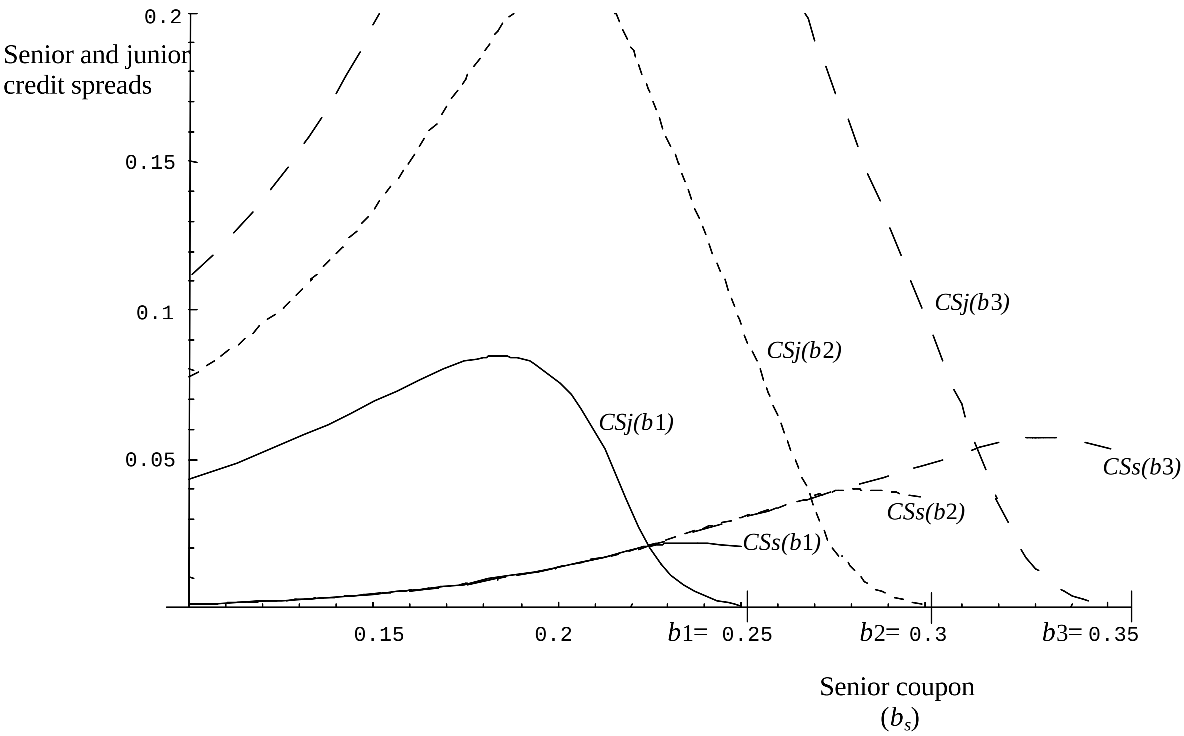

Figure 4. Senior and junior credit spreads for different level of total face value. The senior and junior credit spreads (*CSs* and *CSj*), as functions of b_s, are evaluated at different levels of total coupon *b* but at the same level of cash flows p_t (so they are directly comparable). The total coupon *b* takes values: *b*1=0.25, *b*2=0.30 and *b*3=0.35. Other baseline parameters are as in Figure 3.

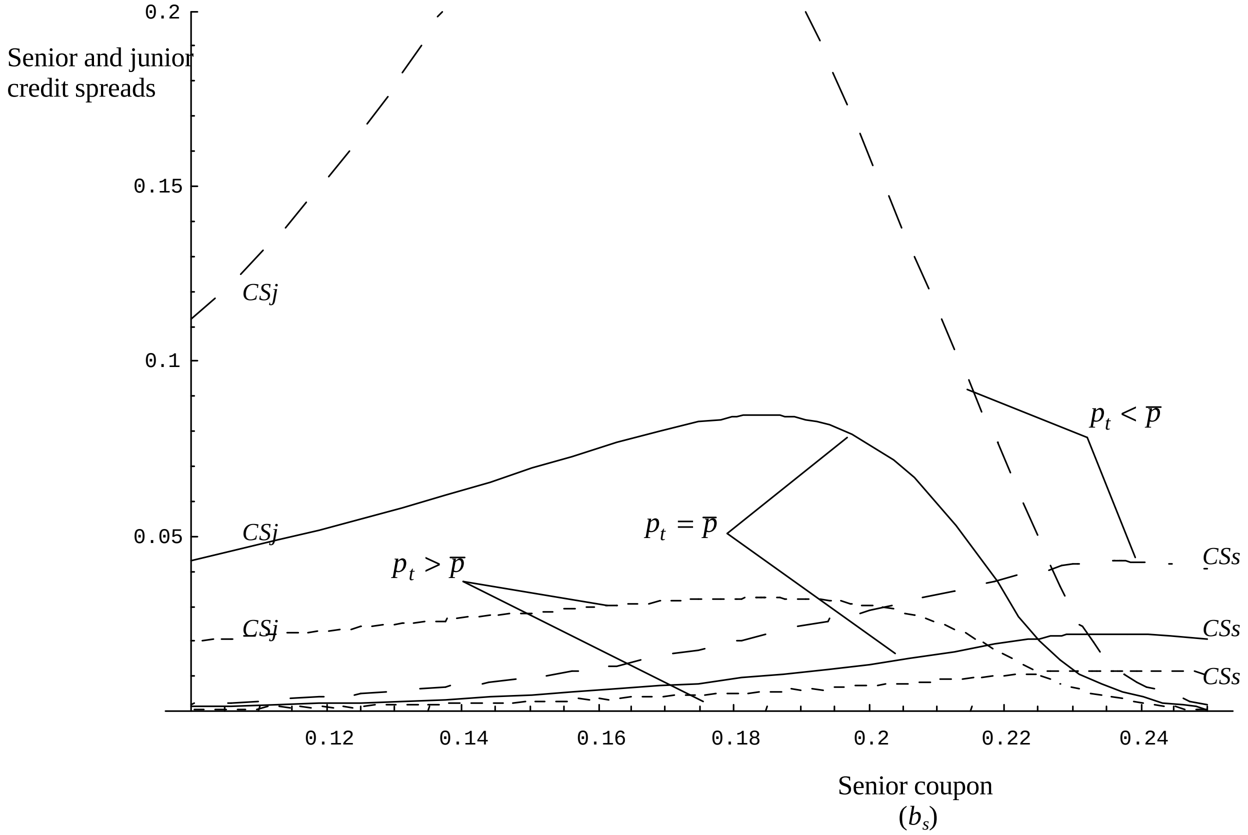

Figure 5. Senior and junior credit spreads evaluated at different level of the cash flow p_{tt}. The senior and junior credit spreads (*CSs* and *CSj*), as functions of b_s, are evaluated at different levels of cash flows p_t, above, equal or below $\overline{p}$. All parameters are as in Figure 3.